

Letters

Linear local tangent space alignment and application to face recognition

Tianhao Zhang*, Jie Yang, Deli Zhao, Xinliang Ge

Institute of Image Processing and Pattern Recognition, Shanghai Jiao Tong University, P.O. Box A0503221, 800 Dongchuan Road, Shanghai 200240, China

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Abstract

In this paper, linear local tangent space alignment (LLTSA), as a novel linear dimensionality reduction algorithm, is proposed. It uses the tangent space in the neighborhood of a data point to represent the local geometry, and then aligns those local tangent spaces in the low-dimensional space which is linearly mapped from the raw high-dimensional space. Since images of faces often belong to a manifold of intrinsically low dimension, we develop LLTSA algorithm for effective face manifold learning and recognition. Comprehensive comparisons and extensive experiments show that LLTSA achieves much higher recognition rates than a few competing methods.

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1. Introduction

The goal of dimensionality reduction is to discover the hidden structure from the raw data automatically. This is also the key issue in unsupervised learning. There are many classical approaches for dimensionality reduction such as principal component analysis (PCA) [9], multidimensional scaling (MDS) [4], and independent component analysis (ICA) [3]. All of these methods are easy to implement and exploited popularly. Unfortunately, they fail to discover the underlying nonlinear structure as traditional linear methods. Recently, more and more nonlinear techniques based manifold learning have been proposed. The representative spectral methods are Isomap [16], locally linear embedding (LLE) [13], Laplacian Eigenmap (LE) [2], local tangent space alignment (LTSA) [17], etc. These nonlinear methods aim to preserve local structures in small neighborhoods and successfully derive the intrinsic features of nonlinear manifolds. However, they are implemented restrictedly on the training sets and cannot show explicit maps on new testing data points for recognition problems. To overcome the drawback, He et al. [5] proposed a

method named locality preserving projection (LPP) to approximate the eigenfunctions of the Laplace–Beltrami operator on the manifold and the new testing points can be mapped to the learned subspace without trouble. LPP is a landmark of linear algorithms based manifold learning.

In this paper, inspired by the idea of LTSA [17], we propose a novel linear dimensionality reduction algorithm, called linear local tangent space alignment (LLTSA). It uses the tangent space in the neighborhood of a data point to represent the local geometry, and then aligns those local tangent spaces in the low-dimensional space which is linearly mapped from the raw high-dimensional space. The method can be viewed as a linear approximation of the nonlinear local tangent space alignment [17] algorithm and the technique of linearization is similar to the fashion of LPP [5]. Since images of faces, represented as high-dimensional pixel arrays, often belong to a manifold of intrinsically low dimension [14], we develop LLTSA algorithm for effective face manifold learning and recognition. Comprehensive comparisons and extensive experiments show that LLTSA achieves much higher recognition rates than a few competing methods.

The rest of the paper is organized as follows: LLTSA algorithm is described concretely in Section 2. In Section 3, several experiments are carried out to evaluate our LLTSA

*Corresponding author. Tel.: +86 21 34204035.

E-mail addresses: zhangtianhao@sjtu.edu.cn (T. Zhang), zhaodeli@sjtu.org (D. Zhao).

algorithm and the experimental results are presented. Finally, the conclusions are given in Section 4.

2. Linear local tangent space alignment

2.1. Manifold learning via linear dimensionality reduction

Consider a data set $X = [x_1, \dots, x_N]$ sampled with noise from M^d which is an underlying nonlinear manifold of dimension d . Furthermore, suppose M^d is embedded in the ambient Euclidean space R^m , where $d < m$. The problem that our algorithm solves is to find a transformation matrix A that maps the set X of N points to the set $Y = [y_1, \dots, y_N]$ in R^d , such that $Y = A^T X H_N$, where $H_N = I - ee^T/N$ represents the centering matrix, I is the identity matrix, and e is an N -dimensional column vector of all ones.

2.2. The algorithm

Given the data set $X = [x_1, \dots, x_N]$ in R^m , for each point x_i , we denote the set of its k nearest neighbors by a matrix $X_i = [x_{i_1}, \dots, x_{i_k}]$. To preserve the local structure of each X_i , we should compute the local linear approximation for the data points in X_i using tangent space [17]. We have

$$\arg \min_{x, \Theta, Q} \sum_{j=1}^k \|x_{i_j} - (x + Q\theta_j)\|_2^2 = \arg \min_{\Theta, Q} \|X_i H_k - Q\Theta\|_2^2, \quad (1)$$

where $H_k = I - ee^T/k$, Q is an orthonormal basis matrix of the tangent space and has d columns, and $\Theta = [\theta_1, \dots, \theta_k]$, where θ_j is the local coordinate corresponding to the basis Q . The optimal x in the above optimization is given by \bar{x}_i , the mean of all the x_{i_j} s and the optimal Q is given by Q_i , the matrix of d left singular vectors of $X_i H_k$ corresponding to its d largest singular values, and Θ is given by Θ_i defined as

$$\Theta_i = Q_i^T X_i H_k = [\theta_1^{(i)}, \dots, \theta_k^{(i)}], \quad \theta_j^{(i)} = Q_i^T (x_{i_j} - \bar{x}_i). \quad (2)$$

Note that, the algorithm just mentioned essentially performs a local principal component analysis; the θ s are the projections of the points in a local neighborhood on the local PCA.

Now, we can construct [17] the global coordinates y_i , $i = 1, \dots, N$, in R^d based on the local coordinate $\theta_j^{(i)}$, which represents the local geometry,

$$y_{i_j} = \bar{y}_i + L_i \theta_j^{(i)} + \varepsilon_j^{(i)}, \quad j = 1, \dots, k, \quad i = 1, \dots, N, \quad (3)$$

where \bar{y}_i is the mean of y_{i_j} s, L_i is a local affine transformation matrix that needs to be determined, and $\varepsilon_j^{(i)}$ the local reconstruction error. Let $Y_i = [y_{i_1}, \dots, y_{i_k}]$ and $E_i = [\varepsilon_1^{(i)}, \dots, \varepsilon_k^{(i)}]$, we have

$$Y_i H_k = L_i \Theta_i + E_i. \quad (4)$$

To preserve as much of the local geometry in the low-dimensional feature space, we intend to find y_i and L_i to

minimize the reconstruction errors $\varepsilon_j^{(i)}$, i.e.,

$$\arg \min_{Y_i, L_i} \sum_i \|E_i\|_2^2 \equiv \arg \min_{Y_i, L_i} \sum_i \|Y_i H_k - L_i \Theta_i\|_2^2. \quad (5)$$

Therefore, the optimal affine transformation matrix L_i has the form $L_i = Y_i H_k \Theta_i^+$, and $E_i = Y_i H_k (I - \Theta_i^+ \Theta_i)$, where Θ_i^+ is the Moore–Penrose generalized inverse of Θ_i .

Let $Y = [y_1, \dots, y_N]$ and S_i be the 0–1 selection matrix such that $Y S_i = Y_i$. The objective function is converted to this form

$$\begin{aligned} \arg \min_Y \sum_i \|E_i\|_F^2 &= \arg \min_Y \|Y S W\|_F^2 \\ &= \arg \min_Y \text{tr}(Y S W W^T S^T Y^T), \end{aligned} \quad (6)$$

where $S = [S_1, \dots, S_N]$, and $W = \text{diag}(W_1, \dots, W_N)$ with $W_i = H_k (I - \Theta_i^+ \Theta_i)$. Note that, according to the numerical analysis in [17], W_i can also be written as

$$W_i = H_k (I - V_i V_i^T), \quad (7)$$

where V_i is the matrix of d right singular vectors of $X_i H_k$ corresponding to its d largest singular values. To uniquely determine Y , we impose the constraint $Y Y^T = I_d$. Finally, considering the map $Y = A^T X H_N$, the objective function has the ultimate form

$$\begin{cases} \arg \min_Y \text{tr}(A^T X H_N B H_N X^T A), \\ A^T X H_N X^T A = I_d, \end{cases} \quad (8)$$

where $B = S W W^T S^T$. It is easily shown that the above minimization problem can be converted to solving a generalized eigenvalue problem as follows:

$$X H_N B H_N X^T \alpha = \lambda X H_N X^T \alpha. \quad (9)$$

Let the column vectors $\alpha_1, \alpha_2, \dots, \alpha_d$ be the solutions of Eq. (9), ordered according to the eigenvalues, $\lambda_1 < \lambda_2 < \dots < \lambda_d$. Thus, the transformation matrix A which minimizes the objective function is as follows:

$$A = (\alpha_1, \alpha_2, \dots, \alpha_d). \quad (10)$$

In the practical problems, one often encounters the difficulty that $X H_N X^T$ is singular. This stems from the fact that the number of data points is much smaller than the dimension of the data. To attack the singularity problem of $X H_N X^T$, we use the PCA [1,6] to project the data set to the principal subspace. In addition, the preprocessing using PCA can reduce the noise.

According to above preparation, we now summarize the Linear Local Tangent Space Alignment algorithm, therefore, steps 3 and step 4 are similar to the corresponding algorithm steps in [17], where we can get the comprehensive reference.

Given a data set $X = [x_1, \dots, x_N]$.

Step 1: PCA projection. Project the data set X into the PCA subspace by throwing away the minor components. To make it clear, we still use X to denote the data set in the PCA subspace in the following steps. We denote by A_{PCA} the transformation matrix of PCA.

Step 2: Determining the neighborhood. For each x_i , $i = 1, \dots, N$, determine the k nearest neighbors x_{i_j} of x_i , $j = 1, \dots, k$.

Step 3: Extracting local information. Compute V_i , the matrix of d right singular vectors of $X_i H_K$ corresponding to its d largest singular values., and set $W_i = H_k(I - V_i V_i^T)$.

Step 4: Constructing alignment matrix. Form the matrix B by locally summing as follows:

$$B(I_i, I_i) \leftarrow B(I_i, I_i) + W_i W_i^T, \quad i = 1, \dots, N \quad (11)$$

with the initialization $B = 0$, where $I_i = \{i_1, \dots, i_k\}$ denotes the set of indices for the k nearest neighbors of x_i .

Step 5: Computing the maps. Compute the eigenvectors and eigenvalues for the generalized eigenvalue problem

$$X H_N B H_N X^T \alpha = \lambda X H_N X^T \alpha. \quad (12)$$

Then we have the solutions $\alpha_1, \alpha_2, \dots, \alpha_d$ ordered according to the eigenvalues, $\lambda_1 < \lambda_2 < \dots < \lambda_d$, and we have $A_{LLTSA} = (\alpha_1, \alpha_2, \dots, \alpha_d)$. Therefore, the ultimate transformation matrix is as follows: $A = A_{PCA} A_{LLTSA}$, and $X \rightarrow Y = A^T X H_N$.

3. Experiments

In this section, several experiments are carried out to evaluate our proposed LLTSA algorithm. We begin with two synthetic examples to show the effectiveness of our method.

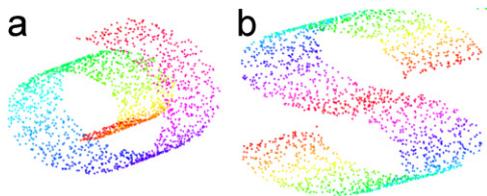


Fig. 1. 2000 random points sampled on the synthetic data. (a) Random points on swiss roll, (b) random points on S-curve.

3.1. Synthetic example

The Swissroll and the S-curve [13], two well-known synthetic data sets, are exploited in the experiment. Fig. 1 shows the 2000 random data sampled on the Swissroll and the S-curve which are used for training. Different from the style of nonlinear methods [17], we can evaluate other novel 2000 random data by the projection which is learned on the training data. Figs. 2 and 3 show the results obtained by PCA, LPP, NPE (linearized version of LLE) [7] and LLTSA in the two data sets. Note that we carry out our LLTSA algorithm without the PCA projection step since the computational problem of eigenanalysis pointed out in Section 2 does not appear. As can be seen, LLTSA together with NPE can preserve the local geometric structure well, whereas PCA and LPP fail to do so.

3.2. Face recognition

A face recognition task can be viewed as a multi-class classification problem. First, we carry out our LLTSA algorithm on the training face images and learn the transformation matrix. Second, each test face image is mapped into a low-dimensional subspace via the transformation matrix. Finally, we classify the test images by the nearest neighbor classifier.

We compare our proposed LLTSA algorithm with baseline, PCA, LPP, NPE [7] and the original LTSA using the publicly available databases: ORL [12], AR [11], and PIE [15]. For the baseline method, the recognition is simply performed in the input space without dimensionality reduction; For the LPP method and the NPE method, the tests are implemented by the unsupervised way. For the original LTSA method, we use a simple interpolation scheme described in [10] to map testing points. For all the experiments, the images are cropped based on the centers of eyes, and the cropped images are normalized to the

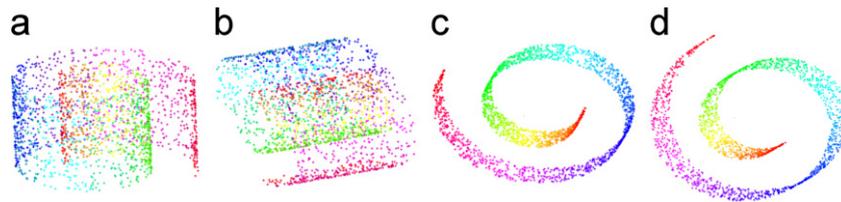


Fig. 2. Results of four methods applied to Swissroll. (a) PCA, (b) LPP, (c) NPE, (d) LLTSA.

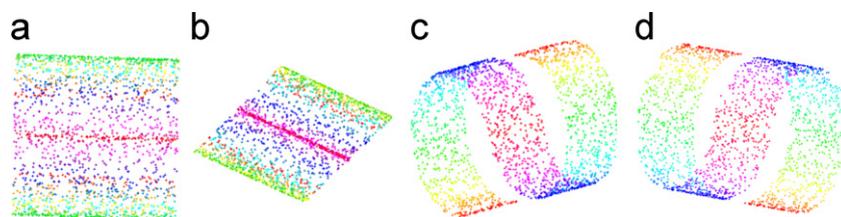


Fig. 3. Results of four methods applied to S-curve. (a) PCA, (b) LPP, (c) NPE, (d) LLTSA.

32 × 32 pixel arrays and with 256 gray levels per pixel. We use the preprocessed versions of the ORL database and the PIE database which are publicly available from X. He’ web page [8]. The images of the AR database are preprocessed by ourselves.

3.2.1. ORL

The ORL database [12] contains 400 images of 40 individuals including variation in facial expression and pose. Fig. 4 illustrates a sample subject of the ORL database along with its all 10 views. For each person, p (3, 5) images are randomly selected for training and the rest are used for testing. For each given p , we average the realizations over 20 random splits and calculate the standard deviations. Fig. 5 shows the plots of the average recognition rates vs. subspace dimensions. The best average results and the standard deviations with the corresponding reduced dimensions are listed in Table 1. As can be seen, our LLTSA method outperforms the other methods involved in this experiment. Note that, it is crucial for LLTSA algorithm to choose the number of neighbors corresponding to different subspace dimensions. Here, we illustrate the point in Fig. 6 which shows the plots of the number of neighbors that yields the best performance as a function of the subspace dimension. We can see that there is an approximate linear relation between the optimal number of neighbors and the subspace dimension. Given the fixed subspace dimension, the linear relation can guide us to find the optimal number of neighbors.



Fig. 4. Sample face images from the ORL database.

3.2.2. AR

The AR database [11] contains 126 subjects (70 men and 56 women) and each subject has 26 face images taken in two sessions separated by 2 weeks time. For each session, 13 face images with varying facial expression, illumination and occlusion were captured. We randomly select 40 different people (20 men and 20 women) from the AR face database. Fig. 7 shows one subject.

Two different strategies of nonoccluded images test and full images test are conducted in this section. For nonoccluded images test, only the nonoccluded images are used. For example, for the subject shown in Fig. 7, only Fig. 7 a through g and n through t added up to 14 face images are used. As for full mages test, all the images including the occluded ones are used. In each strategy, five-fold cross validation is used. Five-fold cross validation means that the sample set is divided into five subsets of approximately equal size. And then the training and testing is carried out five times, each time using one distinct subset for testing and the remaining four subsets for training. Fig. 8 and Table 2 provide the recognition results. We can see that LLTSA method has higher recognition rates than those of other methods. The experimental results also indicate that LLTSA is more robust to the occluded face images.

Table 1
Best recognition rate (%) of six methods on the ORL database

Method	3 Train	5 Train
Baseline	76.89 ± 2.53(1024)	86.26 ± 2.10(1024)
PCA	76.89 ± 2.53(120)	86.26 ± 2.10(188)
LPP	78.14 ± 2.39(55)	88.08 ± 2.09(73)
NPE	80.60 ± 2.12(36)	88.90 ± 1.97(40)
L TSA	78.55 ± 2.14(32)	87.65 ± 2.26(28)
LLTSA	80.89 ± 2.08(35)	89.70 ± 2.10(25)

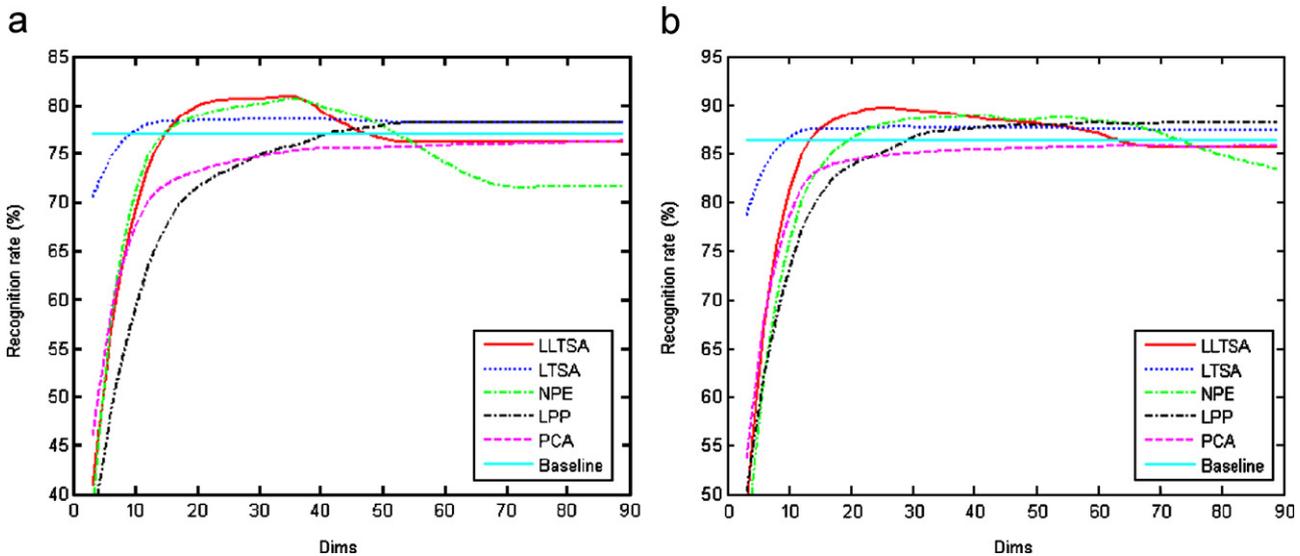


Fig. 5. Recognition rate vs. dimensionality reduction on the ORL database. (a) 3 train, (b) 5 train.

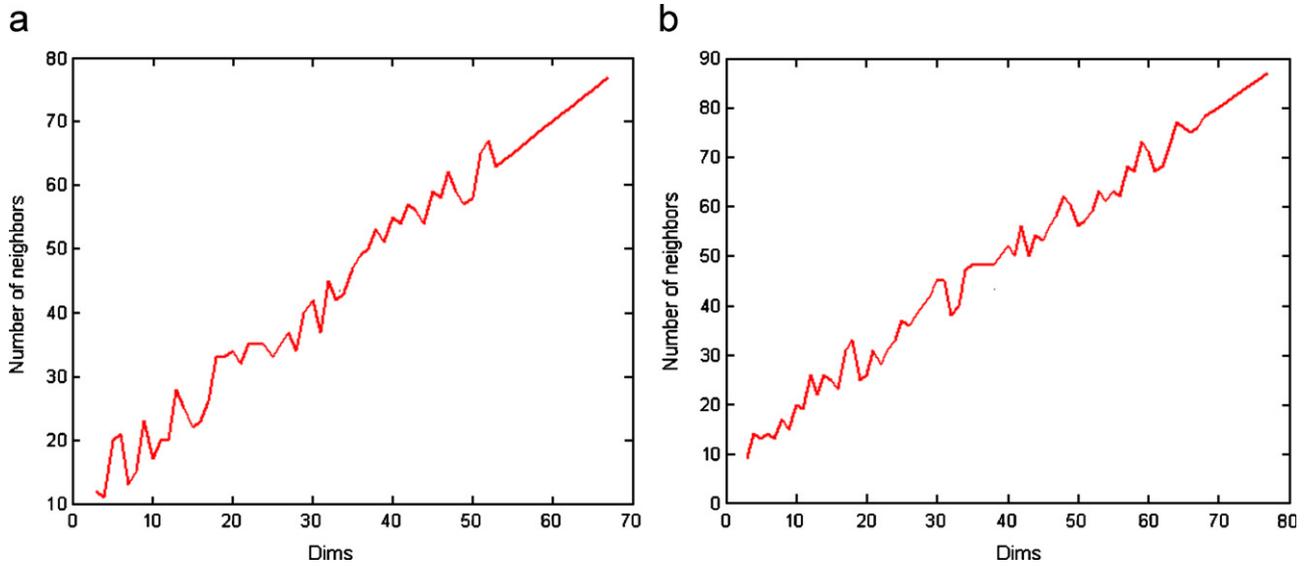


Fig. 6. Number of neighbors vs. dimensionality reduction on the ORL database. (a) 3 train, (b) 5 train.



Fig. 7. Sample face images from the AR database. On the first row are images recorded in the first session, on the second row are recorded in the second session.

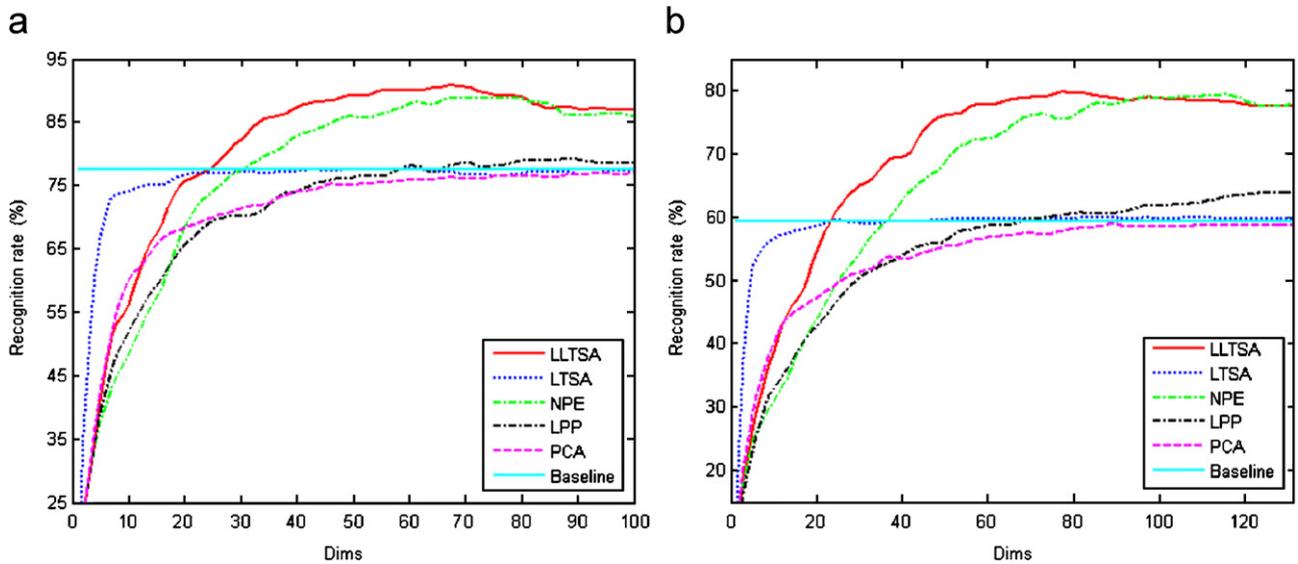


Fig. 8. Recognition rate vs. dimensionality reduction on the AR database. (a) Nonoccluded images test, (b) full images test.

3.2.3. PIE

The PIE database [15] includes over 40,000 facial images of 68 people. We adopt the experimental strategy described

in [6]. One hundred and seventy near frontal face images for each person are employed, 85 for training and the other 85 for testing. Fig. 9 shows several sample images of an

Table 2
Best recognition rate (%) of six methods on the AR database

Method	Nonoccluded	Full
Baseline	77.58 ± 6.08(1024)	59.35 ± 3.52(1024)
PCA	77.58 ± 5.91(293)	59.35 ± 3.52(199)
LPP	79.25 ± 4.68(88)	63.73 ± 4.28(124)
NPE	88.92 ± 2.95(79)	79.30 ± 4.36(116)
LTSA	77.75 ± 6.32(61)	59.87 ± 4.11(108)
LLTSA	90.75 ± 3.00(67)	79.68 ± 4.53(77)



Fig. 9. Sample face images from the PIE database.

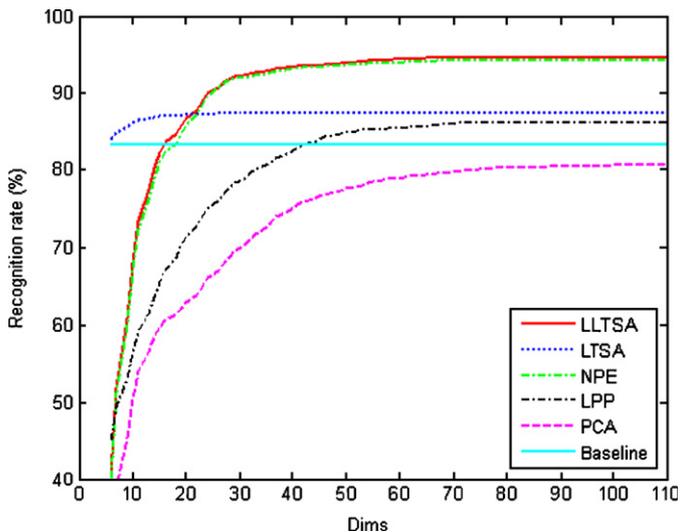


Fig. 10. Recognition rate vs. dimensionality reduction on the PIE database.

individual with different poses, expressions, and illuminations. All the tests are repeated over 20 random splits independently and the average recognition results and the standard deviations are calculated. The recognition results are shown in Fig. 10 and Table 3. Again, LLTSA performs better than the other considered methods.

3.3. Discussions

Several experiments have been conducted on three different face databases. Here, it is necessary to highlight some observations about these tests:

1. As a manifold learning-based method, LLTSA has the ability to detect the intrinsic structure from the raw face data. Moreover, LTSA itself is robust against noises and

Table 3
Best recognition rate (%) of six methods on the PIE database

Method	85 Train
Baseline	83.30 ± 0.84(1024)
PCA	83.30 ± 0.84(1002)
LPP	86.04 ± 0.97(74)
NPE	94.06 ± 0.93(74)
LTSA	87.34 ± 1.01(40)
LLTSA	94.42 ± 0.88(67)

this property is illustrated by Zhang and Zha [17]. LLTSA as the linearization of LTSA inherits this property. So, LLTSA appears robust to the variation in pose, illumination, expression, even to the occlusion case.

2. LTSA uses the local tangent space as a representation of the local structure; LLE preserves the locality based on the linear coefficients of local reconstructions; LE constructs the local graph to model the neighborhood structure. Extensive experiments verified that LTSA and LLE shows better performance on non-linear manifolds unfolding than LE. One can refer to [2,13,17] for figure reviewing. LLTSA, NPE, and LPP, as the linearized versions, have the same conclusions correspondingly, illustrated by the toy examples of Swissroll and S-curve in Section 3.1. LLTSA (LTSA) as well as NPE (LLE) give more precise representations on local structure than that of LPP (LE). It may be the reason why NPE performs comparably to LLTSA to some extent, while LPP performs worse in the experiments of face recognitions. As long as the efficiency in the training step is concerned, however, LPP is competitive. LLTSA needs to estimate the local tangent space at each data point and NPE needs to estimate the linear coefficients for every data point as well. These processes are obviously more expensive than that of LPP.
3. The original LTSA method incorporating interpolation scheme is inferior to LLTSA on recognition rates. The interpolation scheme greatly weakens the ability of LTSA to discover the intrinsic structure, while LLTSA inherits from LTSA better. Furthermore, the task of interpolation requires large memories to store the training sets and searching nearest neighbors is computationally expensive. It is infeasible to implement the nonlinear LTSA method on the costly problems.

4. Conclusions

In this paper, a novel algorithm named (LLTSA) is proposed. LLTSA is a linear dimensionality reduction method, preserving the local coordinates of the data points in the neighborhood with respect to the tangent space. The method can be viewed as a linearization of the LTSA [17] algorithm. The experiments on both synthetic and real face datasets have shown the effectiveness of our developed algorithm.

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Tianhao Zhang was born in Shandong, China, in November 1980. He received the Bachelor's degree in Electrical Engineering from Shandong University in 2002, and the Master's degree in Power Machinery and Engineering from Chang'an University in 2005. Currently, he is a Ph.D. candidate in Institute of Image Processing and Pattern Recognition, Shanghai Jiao Tong University. His research interests include manifold learning, face recognition and computer vision.



Jie Yang was born in Shanghai, China, in August 1964. He received a Ph.D. in computer in Department of Computer, University of Hamburg, Germany. Dr. Yang is now the Professor and Vice-director of Institute of Image Processing and Pattern Recognition, Shanghai Jiao Tong University. He is charged with more than 20 nation and ministry scientific research projects in image processing, pattern recognition, data amalgamation, data mining, and artificial intelligence.



Deli Zhao was born in Anhui, China, in May 1980. He received his bachelor's degree in Electrical Engineering and Automation from China University of Mining and Technology in 2003 and the master's degree from Institute of Image Processing and Pattern Recognition in Shanghai Jiao Tong University in 2006. His current research interests include linear and nonlinear dimensionality reduction and feature extraction.



Xinliang Ge was born in Hebei, China, in 1975. Now he is a doctoral candidate in Institute of Image Processing and Pattern Recognition, Shanghai Jiao Tong University. His research interests include 3D face recognition; facial feature extraction; image processing.