Using a Deformable Surface Model to Obtain a Shape Representation of the Cortex

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Abstract
The problem of obtaining a mathematical representation of the cortex of the human brain is examined. A parametrization of the outer cortex is first obtained using a deformable surface algorithm which, motivated by the structure of the cortex, is constructed to find the central layer of thick surfaces. Based on this parametrization, a hierarchical representation of the cortical structure is proposed through its depth map and its curvature maps at various scales. Various experiments on magnetic resonance data are presented.

I. Introduction
The problem of finding and parametrizing boundaries in two- and three-dimensional images is often an important step toward shape visualization and analysis, and has been extensively studied in the image analysis and computer vision literature. Several methods have been proposed, based both on bottom-up and top-bottom procedures.

One very promising model which combines robustness to noise and the flexibility to represent a broad class of shapes is based on deformable surfaces. Deformable surfaces were introduced in [4] and subsequently explored by several investigators. In [5, 6, 7] non-parametrized deformable surface models were used. Although these models are suitable for the extraction and the visualization of boundaries, they do not provide a mathematical representation of surfaces, which is often required in medical imaging problems involving shape analysis and matching. Several deformable models using parametrized surfaces provide such a mathematical representation, often expressed as a map from the surface to a planar domain where the deformable surface is parametrized. In [4, 8, 9, 10] parametrized deformable surfaces were used, attracted either by edges or feature points.

Our work focuses on the development of a mathematical representation of the structure of the cortex, suitable for automatic feature identification and quantitative analysis. As a first step in achieving this goal we have developed a new deformable surface model. Motivated by the structure of the cortex, which is a thick sheet of fairly uniform thickness enclosing most of the brain tissue, the external forces of our model are constructed to guide the deformable surface toward the central layer of a thick surface. Unlike previous methods which were constructed to find boundaries of objects, our approach can be applied to surfaces of arbitrary thickness, including boundaries which we treat as surfaces of infinitesimal thickness. Moreover, the external forces in our model are robust under the presence of noise and other error sources, since they are determined through an integration as opposed to the differentiation in [4, 9, 10].

The second contribution of our work is in the use of a hierarchical representation of the cortex through its curvature and depth maps, based on the surface parametrization obtained by the deformable surface algorithm. The use of curvature maps for inferring the differential structure of surfaces has been extensively studied in the computer vision literature. Extensive reviews can be found in [12, 13]. In medical imaging, curvature has been used as shape descriptor in [14, 15, 9, 16]. In [14] the minimization of a bending and stretching energy was used to infer the homothetic motion of the boundary of the left ventricle. In [15] the conformal motion of the left ventricle was found based on the Gaussian curvature. Both of these techniques are based on the small deformation assumption since they find the optimal mapping between two surfaces by searching in a local window around each point of the surfaces. In [9] the extrema of the maximum curvature where shown to provide good anatomical landmarks for the skull and were used in [16] for intra- and inter-subject registration.

The convoluted nature of the cortex requires a more complex quantitative description than that of simpler shapes such as the left ventricle of the heart and the skull. In order to obtain a sufficiently rich representation of the cortical structure, in our work we combine various kinds of curvature at various scales. We demonstrate that different curvatures calculated at different scales reflect different anatomical characteristics, which are not reflected all together by a single curvature at a single scale. Moreover, we enrich this representation by introducing an approach for obtaining a depth map of the cortex. The depth map reflects information about the deep foldings of the cortex, in addition to its outer surface topography reflected by the curvature maps. We propose the combination of these different kinds of information into a hierarchical representation of the cortical structure.

In order to facilitate the reading of the paper, in Fig. 10 we provide a brief description of the anatomical terms used in the paper.

II. Deformable Surface Algorithm (DSA)
A. Preliminaries
The cortex is a highly convoluted thick sheet of grey matter enclosing most of the brain. An idealized model for the cortex, which we will use throughout the development of our algorithm, is a thick surface of thickness $\omega$ denoted $C$, with central layer $C(u, v)$. The central layer $C(u, v)$ of $C$ is a closed surface, which is comprised of two open surfaces defined on a unit square and are connected with each other along their common boundary. Consequently, $C(u, v)$ is defined on the domain $D = [0, 2]\times[0, 1]$ and satisfies the appropriate boundary conditions.

Throughout our development we will assume that cross-sectional images of $C$ have been obtained through a tomographic imaging technique and have been preprocessed. Resulting from this preprocessing is a mass function $m(\mathbf{x}) \in [0, 1]$, which is defined in the
data volume $V$, and which represents our confidence that a point $x = (x, y, z) \in V$ belongs to the thick surface.

Often, determining the mass function can be a complex problem by itself. Although there are several automated segmentation techniques which can be used for this purpose [17, 18], manual interaction is often required because of the convoluted nature of the cortex. In our experiments the mass function is determined via threshold-based seeded region growing.

Having described the mass function, we now turn our attention to the deformable surface. It is a closed surface, denoted $\mathbf{x}(u, v) = (x(u, v), y(u, v), z(u, v))$, which lies in $\mathbf{a}(u, v)$ is defined on the domain $D$ and is comprised of two open surfaces connected together along their common boundary. Open surfaces can be readily treated within the same framework by providing the appropriate boundary conditions.

B. Force Model

The deformable surface deforms under the influence of internal and external forces. The external force field is comprised of two components. The first component is derived from the image data and it tends to deform $\mathbf{x}(u, v)$ toward $C$. In order to define this external force we first define $\mu(x)$ to be the mass of $C$ included in a spherical neighborhood $N(x)$ or radius $r$ centered at a point $x \in V$. We also define the center of mass function, $\mathbf{c}(x)$, to be the center of the mass $\mu(x)$ included in $N(x) \cap V$ (see Fig. 1). We now observe that the points lying on $\mathbf{a}(u, v)$ coincide with the center of the mass included in their neighborhoods (see Fig. 1), and therefore they satisfy the following condition:

$$c(a(u, v)) = a(u, v), \quad (u, v) \in D.$$  \hspace{1cm} (1)

Accordingly, we define the external force $F(x)$ acting on a point $x$ to be an attractive force originating from $c(x)$:

$$F(x) = c(x) - x.$$  \hspace{1cm} (2)

The field $F(x)$ acts as a restoring force field tending to bring $\mathbf{x}(u, v)$ back to $C$, and it vanishes on $\mathbf{a}(u, v)$ as (1) and (2) imply.

There are several differences between this external force model and image gradient potential fields [4, 9], which are commonly used in the deformable surface literature. First, it applies to surfaces of arbitrary thickness as opposed to the force models in [4, 9] which apply only to the boundaries of objects. Second, $F(x)$ is based on a local integration embedded in the definition of the center of mass function, in contrast to the differentiation in [4, 9], which makes $F(x)$ robust to noise. Third, since $F$ does not depend on $m(x)$ but only on $c(x)$, it does not cause the accumulation of points of the deformable surface around certain parts of the boundary, unlike external force fields based on the image gradient which can potentially cause the accumulation of points around high gradient parts of the boundary. Fourth, the magnitude of $F$ decreases gradually to zero with the distance of the deformable surface from $\mathbf{a}(u, v)$, unlike, for example, the normalized force model in [9] which has the same magnitude far from and close to the boundary surface.

The second component of the external force field is exerted when $F(x)$ is inactive ($N(x) \cap C = \emptyset$) and is equal to $\gamma(u, v) N(u, v)$, where $N(u, v)$ is the direction of the normal of the surface and $\gamma(u, v)$ is unity when $N(x) \cap C = \emptyset$ and zero elsewhere.

The internal forces acting on the deformable surface are elastic forces that maintain its connectivity during its deformation, and are weighted by a spatially varying elasticity coefficient denoted $\beta(u, v)$, which is equal to a constant $K_0$ when $N(x) \cap C \neq \emptyset$ and equal to a different constant $L_0$ elsewhere. The elasticity parameter $L_0$ determines the degree to which the deformable surface evolves into the cortical gyriations. Moreover, $L_0$ determines the degree to which the deformable surface bridges gaps in the data.\footnote{In the vicinity of a gap in the data, $N(x) \cap C = \emptyset$ and therefore $\beta(u, v) = 0$.}

The objective when selecting $L_0$ is to allow the deformable surface to bridge small gaps while still letting it shrink or expand through larger gaps which might be present because of the underlying brain anatomy\footnote{As we will see in Section IV such an example is the region between the temporal lobes at the bottom of the brain.} as opposed to gaps due to error sources or inter-slice distances.

After a sequence of deformations the deformable surface converges to an equilibrium configuration satisfying the following differential equations:

$$\beta(u, v) \left( \frac{\partial^2 x(u, v)}{\partial u^2} + \frac{\partial^2 x(u, v)}{\partial v^2} \right) + e(x(u, v)) - x(u, v) + \gamma(u, v) N(u, v) = 0,$$  \hspace{1cm} (3)

together with the appropriate boundary conditions.

Equations (3) are discretized and solved iteratively using successive overrelaxation (SOR). The computational requirements are considerably reduced by using a multi-resolution approach with three different sampling densities of the deformable surface.

C. Fixed-Point Algorithm

The deformable surface algorithm provides a parametrization of the outer cortex, and therefore a map from the outer cortex to the plane. However, this map does not necessarily preserve relative distances and, therefore, it can be a distorted image of the cortical anatomy. In order to yield a faithful representation of the cortical structure, DSA must ideally converge to a homothetic map between the cortex and the plane, i.e. an isometric map together with a global scaling factor. In this section we describe an algorithm which seeks such a homothetic map.

Consider a surface parametrized on the unit square by $x(u, v)$. The first fundamental forms of the surface are [11]

$$E = <x_u, x_u>, \quad F = <x_u, x_v>, \quad G = <x_v, x_v>.$$  \hspace{1cm} (4)

Consider, now, two surfaces parametrized by $x(u, v)$ and $x'(u, v)$, and having first fundamental forms $E, F, G$, and $E', F', G'$, respectively. A map which corresponds the point $x(u, v)$ of the first surface to the point $x'(u, v)$ of the second surface is a homothetic map if

$$E = CE', \quad F = CF', \quad G = CG',$$  \hspace{1cm} (5)

where $C$ is a global scaling factor. We now notice that the unit square is parametrized by $x'(u, v) = (u, v, 0)$ and therefore its first fundamental form coefficients are $E' = G' = 1, F' = 0$. Consequently, a parametrization $x$ defines a homothetic map between a surface and the unit square if

$$E = C, \quad F = 0, \quad G = C.$$  \hspace{1cm} (6)
For \( G = 1 \) \( (6) \) defines an isometric map.

In order to find such a map we have developed a fixed-point algorithm, which is an iterative procedure seeking a reparametrization of the deformable surface having first fundamental form as in \( (6) \). We first observe from \( (4) \) and \( (6) \) that the \( u \)-curves (\( u \) fixed) and the \( v \)-curves (\( v \) fixed) of a homothetic map are constant speed curves. Accordingly, in the first two steps of each iteration of our fixed-point algorithm we reparametrize the curves \( \mathbf{x}(u,v) \), \( i = 0, \ldots, N - 1 \), and the curves \( \mathbf{x}(u,v) \), \( i = 0, \ldots, N - 1 \), obtaining constant speed curves. We also observe that \( (4) \) and \( (6) \) imply that the \( u \)-curves and the \( v \)-curves of \( \mathbf{x}(u,v) \) should intersect at right angles. Accordingly, in the third step of each iteration of the fixed-point algorithm the vectors \( \mathbf{x}_u \) and \( \mathbf{x}_v \) are reoriented to form a right angle.

This iterative procedure is repeated until convergence. Although this procedure does not guarantee to find a homothetic map, as we will see in Section IV it yields good solutions in a few iterations.

### III. Curvature and Depth Maps

The deformable surface algorithm described in the previous section parametrizes the outer cortex and maps it through one-to-one nearly homothetic map to the planar domain \( \mathcal{D} \). In order to use this map as a qualitative and quantitative representation of the cortical anatomy, in this section we use two different types of functions: curvature, which reflects the outer surface topography of the cortex, and depth, which reflects the structure of the deep cortical foldings. This representation can serve as the basis for the automatic identification and matching of the cortical features.

#### A. Curvature Maps

From the principles of differential geometry \([11]\), the geometric properties of a surface \( \mathbf{x}(u,v) \) embedded in 3-D are described by the coefficients of its first fundamental form, denoted \( E, F, \) and \( G \) (see \( (4) \)), and by the coefficients of its second fundamental form, denoted \( L, M, \) and \( N \), which are defined below:

\[
L = \langle N, \mathbf{x}_u \rangle, \quad M = \langle N, \mathbf{x}_v \rangle, \quad N = \langle N, \mathbf{x}_{uu} \rangle. \tag{7}
\]

From the first and second fundamental forms, four curvatures can be calculated, which are of interest to our work: the minimum, \( \kappa_m \), the maximum, \( \kappa_M \), the mean, \( \bar{H} \), and the Gaussian, \( G \).

In order to calculate the cortical curvatures at various scales, we use a local least-squares quadratic fit for each of the three discretized functions, \( z(u,v) \), \( g(u,v) \), \( z(u,v) \), which parametrize our deformable surface. By varying the window of this quadratic fit we obtain an estimate of the vector function \( \mathbf{x}(u,v) \) for each \( (u,v) \) at various scales.

The four curvature functions estimated as described above provide complementary information about the characteristics of the cortical shape. In particular, the inward foldings, called sulci, and the outward foldings, called gyri, appear as local maxima and minima, respectively, of the principal curvatures at a fine resolution. Other shape characteristics, such as the overall shape of the temporal lobes and the frontal lobe, appear as curvature extrema of the minimum curvature at low resolution. Finally, the interhemispherical fissure appears as a local maximum of the maximum curvature at medium resolution. Several experiments in Section IV show the curvature maps at various resolutions.

#### B. Depth Map

The curvature maps of the cortex provide information about the outer cortical surface but they yield no information about the depth of the cortical foldings. For example, two sulci that fold differently into the brain can appear to have very similar structure on the outer cortex. However, the folding patterns and the depths of the cerebral sulci are well known to vary considerably from individual to individual. In order to obtain a more complete map of the cortical anatomy, reflecting its deep structure in addition to its outer surface topography, we have introduced a depth estimation algorithm (DEA) described next.

DEA is motivated by the fact that the sulcal foldings are directed along the inward normal of the outer cortical surface. This direction changes toward the deep parts of the foldings, although not drastically. Since the cortex has fairly uniform thickness, and since the cortical convolutions are so sharp that typically the two sides of a sulcus are in contact with each other, the sulcal width is approximately twice the cortical thickness.

Our depth mapping approach incorporates these particular anatomical characteristics of the sulci into a physical model. In this formulation, a number of probe points are initially placed on the outer cortex and let free to move toward the deepest parts of the cortical foldings. The trajectory of each probe point is determined by the action of two force fields (see Fig. 2), under the influence of which the probe points follow an inward trajectory close to the axes of the sulci.

The first force field has a uniform magnitude and its direction is along the inward normal of the cortex at the original position of the probe. The second force field is given by Equation (2). However, the radius of \( \mathcal{N} \) is now \( 2w \) since the effective width of two juxtaposed sides of a sulcus is \( 2w \) (see Fig. 2). A parameter \( \theta \) determines the relative weight between these two forces.

Placed within the above force field, a probe point initially positioned on the outer cortex follows an inward trajectory (see Fig. 2). The length of the probe trajectory is an estimate for the depth of the cortical folding.

The trajectory of a probe is terminated when the magnitude of the total force acting on the probe becomes less than a parameter \( \zeta \in [0,1] \). This typically occurs at the bottom parts of the cortical foldings where \( N \) and \( F \) have almost opposite directions (see Fig. 2).

### IV. Experiments

#### A. Performance of DSA

A set of 80 transaxial magnetic resonance images were acquired. The cortex was then segmented using an interactive region growing algorithm, resulting in a binary mass function equal to one on the cortex and zero elsewhere. The deformable surface was then initialized at a spherical configuration surrounding the brain, and DSA was applied with \( \rho = 3, \ K_0 = 5 \times 10^{-9} \), and \( L_0 = 3 \times 10^{-5} \). The resulting surface is shown in Fig. 3 rendered from two different views.

#### B. Fixed-Point Algorithm

In this experiment we demonstrate the performance of the fixed-point algorithm. A series of 70 transaxial MR images were acquired, and the outer cortical boundary was segmented using seeded region growing and setting \( m(x) = 1 \) at the points where the region
Finally, the rightmost images in Fig. 5 show the result obtained after the grid is almost perfect, which implies that the map from the outer cortex to the curvature was calculated for various window sizes. Fig. 6 shows nine iterations of the fixed-point algorithm; the regularity of the estimation improved and the errors in the normal estimation have been reduced. The irregularity in the spacing of the surface points. The middle images in Fig. 5 show the result after applying three iterations of the fixed-point algorithm; the regularity of the grid has been substantially improved and the errors in the normal estimation have been reduced. Finally, the rightmost images in Fig. 5 show the result obtained after nine iterations of the fixed-point algorithm; the regularity of the grid is almost perfect, which implies that the map from the outer cortex to $D$ is a nearly homothetic map. Moreover, the errors in the estimation of the normal have been entirely eliminated.

C. Curvature and Depth Maps

In this section we first present a series of experiments demonstrating how different curvatures at different resolutions reflect different but complementary aspects of the cortical structure. We then demonstrate the ability of the depth map obtained through DEA to differentiate between sulci with similar superficial but different deep structure.

The surface of the experiment in Section IV.B, shown in Fig. 4, was used in our first experiment in this section. The maximum curvature was calculated for various window sizes. Fig. 6 shows the maximum curvature for LSE window sizes of $6 \times 6$, $10 \times 10$, and $14 \times 14$ points as red superimposed on the rendered surface. In Fig. 6a we can see many of the details of the sulci. Many of these details are lost in Figs. 6b and 6c. However, the details captured at these medium resolutions are typically more consistent across different individuals. Moreover, we notice that information about the width of certain sulci is captured at the resolutions in Figs. 6b and 6c. Most notably, the post-central sulcus of this subject is wide enough to allow the deformable surface to flatten in the middle of the sulcus, resulting in relatively low curvature value for an LSE window of $6 \times 6$. However, a high value is obtained for an LSE window $10 \times 10$ (Fig. 6b) indicating that the post-central sulcus of this individual is much wider than, for example, the central sulcus, which has high value for a smaller window.

Fig. 7 shows the minimum curvature of the deformable surface viewed from the top. Here, a high resolution surface was used, and the minimum curvature was calculated for three different LSE windows. From this figure we see that the minimum curvature reflects the shape of the gyri, the outward folds, in contrast to the maximum curvature which reflected the shape of the sulci, the inward folds. Moreover, we see that with increasing size of the LSE window, only the major gyri are maintained, including the postcentral and precentral gyri, and the superior and middle frontal gyri.

Figure 8b shows the maximum curvature from a bottom view of the brain obtained with a window of $6 \times 6$ points from this low-resolution surface. From this curvature two anatomical characteristics stand out: the cavity created between the temporal lobes, which is where the midbrain structures and the pons and medulla are located, and the Sylvian fissure at the tip of the temporal lobes. These characteristics are found consistently in different brains and therefore are good landmarks for automatic labeling and matching.

Figure 8a shows the minimum curvature of the brain viewed from the bottom. Here, the deformable surface was sampled at low resolution and the minimum curvature was calculated using LSE window of $6 \times 6$ points. It is clear from Fig. 8a that the minimum curvature at low resolution gives a very good outline of the temporal lobes, the occipital lobes, and the frontal lobe, landmarks consistently found across individuals.

In our last experiment we show the performance of DEA. In this experiment each point of the surface in Fig. 9 was treated as a probe point and DEA was applied with $\theta = 1$ and $\xi = 0.95$. The resulting depth map is shown in red in Fig. 9 superimposed on the rendered surface.

Several observations can be made from Fig. 9. First, the interhemispherical fissure can be readily distinguished from any other cortical folding since it corresponds to a sharp local maximum in the depth map$^3$. Second, various sulci which appeared similar in the curvature maps, can be distinguished based on the depth. The most notable difference is that between the post-central sulcus and the central sulcus: the former is deeper, in general, while the latter becomes deep close to the interhemispherical fissure as well as close to the Sylvian fissure. Third, the part of the pre-central sulcus close to the Sylvian fissure is considerably deeper than the rest of the sulci. Finally, the intra-parietal sulci appear to be deeper than most other sulci, with the exception of the post-central sulcus. All these elements provide important information for automatically identifying and matching the cortical structures.

V. Conclusion

An important problem in the automatic analysis of medical image data is the extraction and identification of prominent anatomical features. In this paper, focusing on the human brain, we have presented steps toward that direction by using a deformable surface algorithm together with a hierarchical representation of the cortical structure through its curvature and depth maps.

Our deformable surface algorithm was motivated by the shape of the cortex —a thick surface of fairly uniform thickness— and was designed to find the central layer of a surface of arbitrary thickness. Boundaries of objects were treated within this framework as surfaces of infinitesimal thickness.

Using the outer cortical surface parametrizations obtained through DSA, we calculated different kinds of curvature at different scales, and we showed that they reflect complementary information about the structure of distinct anatomical features. We also introduced a new technique for obtaining a depth map of the cortex. The depth map enriches our hierarchical representation because it provides information about the depth of the cortical foldings in addition to their surface topography reflected in the curvature maps.

$^3$Although for display purposes the maximum value of the depth map was limited, its value at the interhemispherical fissure was much higher than anywhere else.