Finding parametric representations of the cortical sulci using an active contour model

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Abstract
The cortical sulci are brain structures resembling thin convoluted ribbons embedded in three dimensions. The importance of the sulci lies primarily in their relation to the cytoarchitectonic and functional organization of the underlying cortex and in their utilization as features in non-rigid registration methods. This paper presents a methodology for extracting parametric representations of the cerebral sulci from magnetic resonance images. The proposed methodology is based on deformable models utilizing characteristics of the cortical shape. Specifically, a parametric representation of a sulcus is determined by the motion of an active contour along the medial surface of the corresponding cortical fold. The active contour is initialized along the outer boundary of the brain and deforms toward the deep root of a sulcus under the influence of an external force field, restricting it to lie along the medial surface of the particular cortical fold. A parametric representation of the medial surface of the sulcus is obtained as the active contour traverses the sulcus. Based on the first fundamental form of this representation, the location and degree of an interruption of a sulcus can be readily quantified; based on its second fundamental form, shape properties of the sulcus can be determined. This methodology is tested on magnetic resonance images and it is applied to three medical imaging problems: quantitative morphological analysis of the central sulcus; mapping of functional activation along the primary motor cortex and non-rigid registration of brain images.

Keywords: atlas, brain mapping, deformable models, non-rigid registration, sulci

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1. INTRODUCTION

A great deal of attention has been given during the past several years to the quantitative analysis of the morphology of the human brain. Much progress has been made in the development of methodologies for analyzing the shape of subcortical structures (Andreasen et al., 1994; Sclaroff and Pentland, 1994; Martin, 1995; Bookstein, 1996; Davatzikos et al., 1996b; Dean et al., 1996; Haller et al., 1996; Szekely et al., 1996). However, the complexity of the cortical shape has been an obstacle in applying these methods to this highly convoluted structure. In this paper we present a methodology for finding a representation of the shape of the deep cortical folds from magnetic resonance (MR) images, which extends our previously reported work (Vaillant et al., 1996; Vaillant and Davatzikos, 1996).

The region between two juxtaposed sides of a cortical fold, called a sulcus, is a thin convoluted ribbon embedded in three dimensions (3-D). Figure 1 shows the central sulcus, a major sulcus of the human brain, superimposed on cross-sections of an MR volumetric image at three levels. The cerebral sulci are important brain structures, since they are believed to form the boundaries of functionally distinct cortical regions. In particular, during the development of the brain in the embryo, connections between specific cortical regions and connections between cortical regions and subcortical structures, in conjunction with a global and local growth process constrained by the skull, are believed to induce the sharp inward folding of the cortex, resulting in the formation of the sulci. The sulci often demarcate or
are strongly linked with the boundaries between functionally and structurally different cortical regions (Luria, 1966; Rademacher et al., 1993; Mangin et al., 1995). A notable example is that of the central sulcus, which is the boundary between its posteriorly located primary somatosensory cortex, belonging to the parietal lobe and its anteriorly located primary motor cortex, belonging to the frontal lobe. Identifying and quantitatively representing such structure–function links is one of the major goals of the brain map community.

In order to study the cerebral sulci and their relationship with the distribution of the function along their associated cortical folds, we first need to establish a quantitative methodology for describing the shape of these ribbon-like structures. Shape representations of the sulci are a key step toward their quantitative morphological analysis, as well as for determining spatial normalization methods, which are often used in the analysis of functional activation images and for mapping the function of the human brain in standardized coordinate systems. In this paper we present a methodology for finding parametric representations of the sulcal ribbons from magnetic resonance images.

Related to our work is that of Mangin et al. (1995), which studies the cortical topography by determining a graph of the sulci. Although some global characteristics of the sulcal shape, including depth and orientation, were considered in Mangin et al. (1995), no attempt was made to characterize the local geometric structure of the sulci and to determine their average shapes or variability measures. Such measures require parametric, not point-set, representations of the sulci; this is the focus of our work. Also related is the work of Morse et al. (1994) and Pizer et al. (1994), which determines the

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**Figure 1.** The central sulcus superimposed on axial cross-sections of an MR volumetric image at three levels. Only one hemisphere is shown, for clarity.

**Figure 2.** (a) A schematic representation of Van Essen’s folding model: stronger connections (e.g. those between B and C) dominate over weaker connections (e.g. those between B and A), resulting in the formation of a gyrus which minimizes the overall tension. (b) If the folding model of (a) is true, then sulcal interruptions could potentially be formed by a strengthening of connections around a sulcus (e.g. those between B and A). The parametric representation of the sulcal ribbons proposed in this paper allows for the quantitative analysis of such sulcal interruptions and for inter-individual comparisons.
medial axis of arbitrary shapes at multiple scales, the work of Thompson et al. (1996), which is based on manually outlining the sulci and the work of Kruggel (1995) and Le Goualher et al. (1995) which focuses on segmentation of the sulci.

The method we propose in this paper is a physically based algorithm utilizing the particular characteristics of the cortical shape. Specifically, our algorithm uses the outer cortical surface as a starting point. Based on the principal curvatures of this surface, we identify the outer edges of the sulcal ribbons. By placing active contours (Kass et al., 1988; Davatzikos and Prince, 1995) along these edges, which are then set free to move inward following the cortical gyrations, our method obtains a parametric representation of the sulcal ribbons.

There are several advantages in representing a sulcus as a parameterized surface. First, the shape of a sulcus is always described with reference to a standard domain, the parametric domain. Therefore, inter-subject comparisons as well as pooling and averaging of data are straightforward. Secondly, a map from a sulcus in an individual brain to the same sulcus in another brain can be established much more easily by first mapping each sulcus to a common domain, the parametric domain.

The most important advantage of the parametric representation of the sulci proposed herein, however, is that it can quantify the location and extent of interruptions. Interruptions of the sulci have received a great deal of attention recently (Ono et al., 1991; Mangin et al., 1995; Demliau et al., 1996), since they are closely associated with white matter connections between adjacent gyri. The importance of sulcal interruptions is further bolstered by the recent theory of cortical folding proposed by Van Essen (1997), according to which the formation of gyri and sulci is caused by competing tension-generated forces applied by white matter tracks connecting adjacent cortical regions. Strong white matter connections dominate over weak ones and result in the formation of gyri, the juxtaposed sides of which are strongly connected (see Figure 2a). If weaker connections running along the peri-sulcal cortex become strong enough, then they could potentially cause the formation of additional cortical gyrations, resulting in the partial or total interruption of a sulcus (Figure 2b). The location and depth of such interruptions is quantified by the first fundamental form of the parametric representation of the sulcal ribbons proposed herein, as we describe in detail in Subsection 2.6.

Three main applications of the parameterizations of the sulci are considered in this paper. The first is the quantitative morphological analysis of the sulci. In particular, we use the parametric representation of the sulcal ribbons to determine the average shape of a sulcus and the inter-individual variability around this average. Inter-subject comparisons can be readily formulated based on this representation. We also demonstrate the relation between the first fundamental form of the parametric representation of an interrupted sulcus and the interruption itself. The second application is in mapping the cortical function along the motor cortex, which is examined in Subsection 3.2. Finally, the third application is in the spatial normalization and registration of brain images, in which a key issue is often obtaining a parametric representation of distinct surfaces which can then be matched based on their geometric characteristics (Sandor, 1994; Davatzikos, 1996a, b; Thompson and Toga, 1996). The sulcal ribbons of distinct cortical folds can be used for this purpose in shape-transformation methodologies (Davatzikos, 1996b; Thompson and Toga, 1996).

2. EXTRACTION OF THE SULCAL RIBBONS

Despite the remarkable similarity of the sulcal patterns across individuals, a substantial variability is notable. Except for a few major sulci, such as the central sulcus and the Sylvian fissure, most secondary and tertiary sulci differ in topology (Ono et al., 1991; Mangin et al., 1995). In particular, most sulci are frequently interrupted in part or in whole by cortical gyri and some consist of a variable number of components (e.g. the pre-central sulcus).

In our work, we model a sulcus as the collection of a number of different components or sulcal ribbons. Each ribbon, which may or may not be interrupted, is parameterized in the unit square; the use of a common parametric domain facilitates inter-subject comparisons. For example, the sulcus shown in Figure 3 comprises two ribbons. The first ribbon consists of pieces 1 and 2 (1-D analogs of the sulcal ribbons are shown here for simplicity). At the interruption between 1 and 2, the sulcal ribbon is assumed to have zero depth. Since these two pieces are part of the same ribbon, they are mapped jointly to the unit square, proportionally to their extent. As we will see in Subsection 2.6, the degree to which a gyrus between 1 and 2 interrupts the sulcal ribbon is expressed quantitatively by the first fundamental form coefficients of a parameterization of the ribbon. The second ribbon, 3, is also parameterized in the unit square; this ribbon is treated separately, since it intersects the ribbon formed by 1 and 2.

The main contribution of this paper is a methodology for obtaining a parametric representation of an individual sulcal ribbon, interrupted or not, from MR images.

2.1. Overall idea

The overall idea behind our algorithm is based on the particular shape of the sulcal ribbons. Specifically, a parametric representation of a sulcal ribbon is extracted from an MR volumetric image as an active contour slides along the sulcal groove in between two juxtaposed sides of the associated
using a deformable surface algorithm described elsewhere
find a parametric representation of the outer cortical surface
(principal) curvature along this surface. Specifically, we first
ations along the outer cortical surface, using the minimum
ration of the active contour, we first find the sulcal inden-
ary of the brain. In order to determine this initial configu-
each sulcal ribbon is initially placed along the outer bound-
2.2. Initial placement of the active contours
As mentioned earlier in the paper, one active contour for
sulcus. The second one resembles a gravitational force and
motion of the active contour to be along the medial surface of
the root of the sulcus.
Figure 4. A schematic diagram of the force field acting on the active
contour. The outer cortical contour (solid curve) represents a cross-
section of the outer cortical surface. The dashed curve represents
the trajectory of a point belonging to an active contour oriented per-
pendicularly to the image plane. \( F_1 \) attracts the active contour point
along the medial axis of a sulcus. \( F_2 \) guides the active contour point
inwards to the root of the sulcus.
cortical fold (see Figure 4). The collection of the deformed
configurations of the active contour readily provides a paramet-
erization of the sulcal ribbon. The initial active contour is
along the outer cortical surface; its final configuration is along
the root of the sulcus.
The inward motion of the active contour is determined by
two force fields (see Figure 4). The first one restricts the
motion of the active contour to be along the medial surface of
a sulcus. The second one resembles a gravitational force and
is responsible for the inward motion of the active contour.

2.2. Initial placement of the active contours
As mentioned earlier in the paper, one active contour for
each sulcal ribbon is initially placed along the outer bound-
ary of the brain. In order to determine this initial configu-
ration of the active contour, we first find the sulcal inden-
tations along the outer cortical surface, using the minimum
(principal) curvature along this surface. Specifically, we first
find a parametric representation of the outer cortical surface
using a deformable surface algorithm described elsewhere
(Davatzikos and Bryan, 1995, 1996). In this algorithm, a de-
formable surface, initially having a spherical configuration
surrounding the brain, deforms like a contracting elastic mem-
brane attracted by the outer cortical boundary and eventually
adapts to its shape.
We determine the outer cortical boundary through a se-
quence of three operations. Morphological erosion is first
used to detach the brain tissue from the nearby dura, skull
and bone marrow. A growing seeded region then extracts
the brain tissue. Finally, morphological dilation recaptures
the brain tissue lost in the erosion step. All these operations
are performed in 3-D. Manual editing of the final result is
often required if the erosion step does not fully detach the
parenchyma. Figure 5 shows a representative cross-section
from a 3-D MR image and the result after the skull-stripping
procedure.
The deformable surface algorithm is applied to the skull-
stripped images. Resulting from this algorithm is a paramet-
ric representation of the outer cortical surface, denoted by
\( b(u, v) \), where \((u, v)\) takes values in a planar domain in which
the deformable surface is parameterized. This surface does
not go deep into the cortical folds—it only follows the outer
cortical indentations. These indentations are the outer edges
of the sulci and they are identified along the outer cortical
surface using the minimum (principal) curvature (Millman
and Parker, 1977) of \( b(u, v) \), which has a high value along
them (Davatzikos and Bryan, 1995, 1996) (see Figure 11). On
these high-curvature curves we then initialize active contours
having evenly spaced points. The initial configuration of an
active contour will be denoted by \( x(s, 0) = x(s, t)_{t=0} \). Here,
the parameter \( t \in [0, 1] \) denotes time. As \( s \in [0, 1] \) sweeps
the unit interval, \( x(s, t) \) runs along the active contour at its
configuration at time \( t \). Equivalently, \( x(s, t) \) and \( x(s, t + \Delta t) \)
are two consecutive deformed configurations of the active
contour.

2.3. External force fields
After its initial placement along the outer edge of a sulcal rib-
on, the active contour slides along the medial surface of the
sulcus, in between two opposite sides of a cortical fold, under
the influence of its internal elastic forces and two external
force fields which are described next.

Center-of-mass force
The first force field, \( F_1 \), acting on the active contour restricts
its motion to be on the sulcal medial surface. This force
is based on the observation that the points lying on the me-
dial surface of a sulcus satisfy the following condition (see
Figure 6):

\[
X = c(X),
\]
Parametric representations of the cortical sulci

\[ X = c(X) \neq c(X) \]

Figure 5. (a) A cross-section from a 3-D MR image and (b) the result after extracting the parenchyma.

Figure 6. Schematic diagram of the cross-section of a sulcus and its surrounding cortex. \( c(X) \) is the center of the cortical mass included in a circular neighborhood around \( X \). If \( X \) is not on the medial surface, it moves towards it under the influence of the force \( c(X) - X \).

where \( c(X) \) is the center of the cortical mass included in a spherical neighborhood, \( N(X) \), centered on \( X \). Accordingly, we define the force acting on a point located at \( X \) as

\[
F_1(X) = \frac{c(X) - X}{\rho(X)},
\]

where \( \rho(X) \) is the radius of \( N(X) \) and is spatially varying, as described below. We note that the center of mass force described in (2) allows for the use of either hard or soft segmentation. The latter results in more robust behavior of the algorithm in the presence of noise and partial-volume averaging, since the motion of the active contour is affected mostly by the points that are most likely to belong to the gray matter.

If the cortex had exactly uniform thickness throughout its extent, and if the two juxtaposed sides of the cortical folds were always in contact with each other, then \( \rho \) in (2) would be fixed to a value equal to the cortical thickness. In that case, \( F_1 \) would be exactly zero on the sulci. However, this is often not the case. In order to account for variations in the cortical and sulcal thickness, we allow \( \rho(X) \) to vary throughout the cortex; at each point \( X \), \( N(X) \) adapts its size to encompass the cortical gray matter in its full width. In particular, at each point \( X \), \( \rho(X) \) is defined as the radius of the smallest spherical neighborhood intersecting the white matter on both sides of a cortical fold:

\[
\rho(X) = \min \{ \rho : \text{size}(N(X) \cap \text{white matter mass}) > \text{threshold} \}.
\]

We typically set the threshold above to 12, i.e. the equivalent of 12 white matter voxels.

Now consider an active contour point located at \( X \). If \( X \) is exactly on the medial axis of the sulcus then \( F_1(X) = 0 \); in this case \( F_1 \) does not affect this point. Otherwise \( F_1 \) moves the point towards the sulcal medial axis (see Figure 6). After a sequence of incremental movements, the point balances at a position satisfying (1) (within some preset tolerance factor) where \( F_1 \) vanishes. It is important to note that \( \rho(\cdot) \) adapts continuously to the cortical thickness during this restoring motion. At equilibrium, the spherical neighborhood balances between the two opposite boundaries of the cortical fold.

\[ F_1(X) = \frac{c(X) - X}{\rho(X)} \]

\[ \rho(X) = \min \{ \rho : \text{size}(N(X) \cap \text{white matter mass}) > \text{threshold} \} \]

\[ F_1(X) = \frac{c(X) - X}{\rho(X)} \]

\[ \rho(X) = \min \{ \rho : \text{size}(N(X) \cap \text{white matter mass}) > \text{threshold} \} \]
Inward force

The second force field, $F_2$, acting on the active contour is responsible for its inward motion toward the root of a sulcus. At the active contour’s initial configuration $x(s, 0)$, $F_2$ at each contour point is in the direction of the inward normal to the outer cortical surface, which is denoted by $F_N[x(s, 0)]$. As mentioned earlier, the orientation of the sulci tends to deviate from this normal direction, especially deep in the brain. Accordingly, $F_2$ is constantly adjusted based on the dynamics of the inward motion of the active contour. Specifically, $F_2$ is defined as

$$F_2(s, t) = -\alpha x_t - \beta x_{tt} + \gamma F_N[x(s, 0)],$$  

where subscripts denote partial derivatives. The first two terms in (3) give rise to damping and inertial influences respectively, and have the effect of averaging the previously traveled direction of the active contour with the initial inward direction, $F_N[x(s, 0)]$. This actively adapts the inward force to the shape of the sulcus.

Finally, a third force acting on each point of the active contour is equal to $K x_{ss}(s, t)$, where $K$ is a parameter determining the elasticity of the active contour. After discretization, this term represents the sum of two spring-like forces, originating from its two neighbors.

Under the influence of these force fields the active contour deforms elastically, sliding along the medial surface of a sulcus towards its deepest edge. The inward trajectory of each active contour point is terminated when the magnitude of the total force acting on it becomes less than a threshold $\zeta \in [0, 1]$. The total force acting on each active contour point is the sum of $F_1$, $F_2$ and the elastic forces from its neighbors. The termination of the active contour motion typically occurs at the bottom parts of the cortical folds where $F_1$ and $F_2$ have almost opposite directions (see Figure 4).

2.4. Refinement of the sulcal surface

The collection of the deformed configurations of the active contour constitute a surface, parameterized by $x(s, t)$, which is defined in the parametric domain $[0, 1] \times [0, T_{\text{max}}]$, where $T_{\text{max}}$ is the total number of time steps. A uniform resampling of this surface results in a parameterization in the unit square $\mathcal{U}$. This surface tends to be smooth along one family of isoparametric curves, the one obtained by fixing $t$, because of the internal elastic forces of the active contour which are along these curves. However, it is not necessarily smooth along the other family of isoparametric curves, the one obtained by fixing $s$, which are oriented along the direction of the inward motion, because of the ‘jerkiness’ of the active contour motion. In order to obtain a smooth surface, $\hat{x}(s, t)$, from $x(s, t)$, we treat $x(s, t)$ as an initial estimate of the parametric grid of the sulcus. This initial estimate is then refined like an elastic membrane, under the influence of the center-of-mass external force field. The final ribbon is defined as follows:

$$\hat{x}(s, t) = \arg\min_{y(s, t)} \int \|y(s, t) - c[y(s, t)]\|^2 \, ds \, dt$$

$$+ L \int \left(\|y_x(s, t)\|^2 + \|y_y(s, t)\|^2\right) \, ds \, dt,$$  

where subscripts denote partial derivatives. Fixed boundary conditions are applied:

$$\hat{x}(s, t) = x(s, t), \quad (s, t) \in \mathcal{B}(\mathcal{U})$$

where $\mathcal{B}(\mathcal{U})$ is the boundary of the unit square.

Equation (4) is discretized and solved iteratively, resulting in a smoother surface parameterized by $\hat{x}(s, t)$, which is defined in the same domain $\mathcal{U}$. Figure 7 illustrates the effect of this procedure. Specifically, Figure 7a shows the surface, $x(s, t)$, that resulted from the inward motion of the active contour and Figure 7b shows the corresponding smoothed surface, $\hat{x}(s, t)$.

2.5. Fixed-point algorithm

The solution of (4) is one possible parameterization of a sulcal ribbon. Although there is an infinite number of such parameterizations of a sulcus (or of any surface, as a matter of fact), any of them is adequate for visualization purposes, since the shape of a surface does not depend on the way that the surface is parameterized. However, for all the applications considered in the following section, it is at least desirable, if not necessary, to find a unique parameterization with specific properties. This is especially important in a procedure determining

![Image](image-url)
the average shape of a sulcus in which all the sulci whose average form is sought must be parameterized in a consistent way, so that corresponding points are averaged together. For example, Figure 8 shows two surfaces, corresponding to two hypothetical sulcal ribbons. The arrows show corresponding points, i.e. points with the same \((s, t)\) parametric coordinates. The surfaces in (b) and (c) have identical shapes, but clearly the parameterization in (b) is a much more reasonable match to that in (a). We note that if any corresponding features along two homologous sulci are available, then such features can be used to stretch locally the grid in Figure 8b, so that the features are in register, i.e. they have the same parametric coordinates \((s, t)\). In Subsection 3.3 we expand upon this idea.

One way to obtain a consistent surface parameterization across subjects, in the absence of features along the sulcus, is to find a parameterization \(x(s, t)\) that has the following first fundamental form (Millman and Parker, 1977):

\[
E = \lambda_s, \quad F = 0, \quad G = \lambda_t
\]

where \(\lambda_s\) and \(\lambda_t\) are constants. Intuitively, such a parameterization can be obtained by stretching the parametric domain \(U\) by a factor of \(\lambda_s\), horizontally, a factor \(\lambda_t\), vertically and folding it in 3-D while preserving the angles between its isoparametric curves. If \(\lambda_s = \lambda_t\), then the parameterization is homothetic, i.e. an isometry together with a uniform scaling factor.

Equation (5) implies that the surface with this first fundamental form must have isoparametric curves of constant speed (although the speeds of the \(s\)-curves and the \(t\)-curves differ if \(\lambda_s \neq \lambda_t\)). Moreover, the isoparametric curves intersect at right angles. Such a parameterization does not exist for an arbitrary surface. However, for ribbon-like structures a parameterization that is very close to that in (5) can be found through an iterative procedure that sequentially reparameterizes each isoparametric curve by a constant speed parameterization. In Davatzikos and Bryan (1996) we describe this procedure in more detail. This iterative procedure, which we use herein, typically converges in a few iterations.

2.6. Sulcal interruptions and the first fundamental form

One important aspect of the shape of the sulci is the pattern of interruptions of the sulcal ribbons across individuals. Such interruptions are caused by white matter tracks connecting adjacent gyri and can potentially elucidate several aspects of the connectivity between different cortical regions of the brain. Ono et al. (1991) measured the frequency and the number of interruptions of the cerebral sulci. Although this representation provides a rough picture of the sulcal patterns, it does not precisely quantify the extent of partial interruptions or their location. Moreover, it does not lend itself to inter-subject comparisons, except for comparisons of measures reflecting the frequency of total or partial interruptions.

Our parameterization of the sulcal ribbons in a standard reference domain, namely the unit square \(U\), regardless of the number of interruptions and the depth of each interruption, facilitates and unifies the quantitative analysis of the shape and interruption pattern of the cerebral sulci. Specifically, a partial or total interruption of a sulcus causes the ‘squashing’ of the isoparametric curves of a sulcal parametric representation, \(x(s, t)\), in the vicinity of the interruption. This is shown schematically in Figure 9. This squashing results in low values of one of the first fundamental form coefficients,
in particular of \( G \), which is given below:

\[
G = (x_s, x_t).
\]

A complete interruption will result in \( G = 0 \).

In addition to quantifying the degree of a sulcal interruption, the first fundamental form coefficient \( G \) also measures the depth of a sulcus at each point along its long axis. Similarly, the coefficient \( E = (x_s, x_t) \) measures the length of the sulcus for each depth level. It is important to note that these two coefficients of the first fundamental form can readily be compared across subjects, since they are always defined in the same domain. Moreover, they can be correlated with functional activation measurements; such measurements can potentially reveal links between sulcal interruptions, connecting neighboring cortical regions and a concurrent activation of such cortical regions. Finally, the second fundamental form coefficients can also be readily calculated from the parametric representation \( x(s, t) \); these parameters reflect intrinsic and extrinsic shape properties of the sulcal ribbons.

3. APPLICATIONS OF SULCAL PARAMETERIZATION

3.1. Morphological analysis of the sulci

One of the fundamental problems in the emerging field of computational neuroanatomy is determining average shapes of brain structures and measuring their inter-individual variability (Evans et al., 1993; Miller et al., 1993; Bookstein, 1996; Davatzikos et al., 1996b; Dean et al., 1996; Thompson et al., 1996). The parametric representation of the shape of the sulci presented in the previous section can form the basis for a shape analysis of the sulcal structure and for determining average sulcal shapes. In order to obtain the average of a particular sulcus in a population, we apply the Procrustes fit, described in more detail in Bookstein (1996). In this algorithm, the sulci are first translated to have a common centroid. Then they are scaled so that the sum of the squared magnitudes of the vectors from the centroid of each sulcus to the points along the sulcus is equal to unity. Subsequently, the sulci are rotated iteratively until their distance from their average becomes minimal. This is a very fast algorithm, so typically a few iterations are sufficient for convergence.

If the resulting sulcal parameterizations of \( N \) subjects in a group after the Procrustes fit are \( p_1(s, t), p_2(s, t), \ldots, p_N(s, t), (s, t) \in \mathcal{U} \), then the average sulcus is

\[
\bar{p}(s, t) = \frac{1}{N} \sum_{i=1}^{N} p_i(s, t).
\]

It is important to apply the Procrustes fit before averaging. This is because of the highly convoluted nature of the cortical folds. In particular, positional or rotational differences between convoluted shapes result, in general, in an average shape that is smoother than each individual shape, even in the extreme case that all averaged shapes are identical. The Procrustes fit minimizes this undesirable effect of rigid-body displacements.

Obtaining a measure of the variance of a sulcus at a particular location with parametric coordinates \( (s, t) \) or the covariance tensor around the average shape requires special considerations, since such measurements might inevitably depend on the specific parameterization scheme used to represent the shape of each sulcus. In particular, one can locally stretch or shrink the parametric grid of a sulcus around \( (s, t) \) in the subjects of the population and subsequently measure a higher variance at those parametric coordinates, as shown schematically in Figure 10. The apparent increase in the variance in such a case would be due to a component along the tangent plane of each sulcus at \( (s, t) \), corresponding to the stretching or shrinking imposed by the parameterization.

Our confidence in the measurement of variability perpendicular to the tangent plane at each point of a sulcal ribbon is high, because the extraction of the sulcal ribbon is guided by image properties that clearly indicate where the sulcus is. However, our confidence on the specific parameterization of a sulcal ribbon is much lower, since no image information is available to guide the placement of a parametric grid on the sulcal surface. Although our parameterization, which consists of a uniform stretching of the parametric grid along the ribbon, is a sensible choice in the absence of specific features along the sulcus, it is still one of many possible ones.

In order to minimize the effects of the parameterization on the measurement of the variance around the average shape,

![Figure 9. A schematic diagram showing the vertical ‘squashing’ of one family of isoparametric curves in the vicinity of an interruption. The first fundamental form coefficients of this parameterization quantify the amount of squashing of the isoparametric curves and therefore the depth of the interruption.](image-url)
of a sulcus, in the experiments of Section 4 we decomposed the residual vector \( \mathbf{p}(s, t) - \bar{\mathbf{p}}(s, t) \) into two components, one perpendicular to the tangent plane of each sulcus at \((s, t)\) and one along it. We measured the variance of the component perpendicular to the tangent plane of each sulcus, which is denoted by \( p_n(s, t) \). This component depends primarily on the shape of the sulci and to a much lesser extent on the way they are parameterized. We note that a similar decomposition cannot be made in obtaining the average shape, nor is it as necessary as it is in determining the variability of a sulcal shape. This is because tangential variations in the parameterization of each sulcus that are randomly distributed around our uniform-stretching parameterization leave the average unaffected, while they increase the variability around the average. For example, the average of points \( A' \) and \( B' \) in Figure 10 is the same as the average of the points \( A \) and \( B \), but the lengths of the residual vectors \( OA' \) and \( OB' \), which determine the variance, are larger than those of \( OA \) and \( OB \).

### 3.2. Functional mapping

The way in which a sulcal ribbon folds in 3-D can vary considerably across individuals. However, the distribution of function along the surrounding cortex is believed to be more consistent\(^a\). This suggests that it may be beneficial to perform the analysis of functional data pertaining to the cortical folds in a coordinate system that is independent of the way in which the sulci fold in 3-D, rather than in a standard 3-D Cartesian coordinate system. Our parameterization readily provides such a coordinate system: the 2-D Cartesian coordinate system defined on the planar domain \( \mathcal{U} \). The analysis of a function along either side of a sulcus in this domain is independent of the embedding of a sulcus in 3-D.

Performing the analysis of functional data on the domain \( \mathcal{U} \) can improve the accuracy of the widely used functional imaging statistical parametric mapping methods, particularly the ‘SPM’ method (Friston et al., 1995). This is because SPM performs a statistical analysis of the activation signal on a slice by slice basis. However, such an analysis inevitably depends on the way that a cross-sectional image intersects the cortical surface and therefore it depends on the way the latter is embedded in 3-D. If, however, the functional signal from the peri-sulcal cortex is mapped to \( \mathcal{U} \), then functional activity can be analyzed on a single ‘slice’ spanning the whole sulcus (e.g. see Figure 21).

A final advantage of mapping functional image data onto the domain \( \mathcal{U} \) is that the functional activity of the surrounding cortex, which now can be presented as a 2-D image, can easily be visualized, instead of being mentally reconstructed from cross-sectional images. For example, the somatotopic distribution of a function along the entire primary motor and somatosensory cortices can be visualized by looking at the distribution of functions within \( \mathcal{U} \).

### 3.3. Non-rigid registration of 3-D images

Finding parametric representations of the sulcal ribbons also finds application in the spatial normalization and registration of brain images. We have previously reported a methodology (Davatzikos, 1996a, b, 1997) that elastically warps one brain image to another, by mapping a number of surfaces in one brain to their homologous surfaces in the other brain. In Davatzikos (1996a, b, 1997) the warping was based on the outer cortical and the ventricular surfaces, which can be extracted fairly easily from MR images. In that approach, the parenchyma of one brain is mapped to the parenchyma in another brain; moreover, a curvature-based matching along the outer cortical surface brings individual cortical regions into register. In Davatzikos (1996a, b, 1997), however, the problem of registration of the deep cortical folds was not addressed. In this paper we utilize the sulcal surfaces as additional features driving the 3-D elastic warping. Related are the approaches of Thompson and Toga (1996), which use a different warping transformation and require the manual outlining of the sulci, the approach of Collins et al. (1996) which utilizes voxel-based representations of the cortical sulci and the approach of Declerck et al. (1995) which addresses the issue of sulcal registration along the outer cortical surface.

Let \( x_1(s, t) \) and \( x_2(s, t) \) be the parameterizations of two homologous sulci. In our elastic warping algorithm we deform one sulcus to the other so that

\[ x_1(s, t) \rightarrow x_2(s, t), \quad \forall (s, t) \in \mathcal{U}. \quad (7) \]

This map clearly results in point correspondences that are identical whether sulcus A is mapped to sulcus B or sulcus B

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\( ^a \) A notable example is the somatotopic projection of sensory and motor function along the central sulcus.

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Figure 10. A 1-D analog of two ribbons and their average (each curve is the 1-D analog of a ribbon). The apparent variability of these curves around their average at \( O \) is significantly higher if the curves are parameterized slightly differently \( (A \rightarrow A', B \rightarrow B') \); the average position of \( O \) does not change if the variations of the parameterizations of the two curves along their tangents are randomly distributed.
is mapped to sulcus A. This is because the point correspondences are determined by the parameterizations of two homologous sulci, as in (7), and are therefore independent of the warping transformation itself. The map in (7) effectively stretches one sulcus uniformly along each family of isoparametric curves so that it matches the other, while maintaining the angles at which the horizontal and vertical isoparametric curves intersect. The rest of the 3-D image deforms following the equations of an elastic solid deformation, which attempts to bring the sulcal ribbons, the outer cortical boundary and the ventricular boundary into spatial coincidence between the warped and the target brains.

Non-uniform sulcal stretching

In the absence of either anatomical or functional landmarks, the directionally uniform stretching described in Subsection 2.5 is the best guess for defining point-to-point correspondences between two sulci. However, often the structure of the sulci allows us to obtain a more accurate map from one sulcus to another. A notable example is the central sulcus, which has a fairly consistent folding pattern forming an S-like shape (Ono et al., 1991). This folding pattern is reflected by the principal curvatures along the sulcus, which can therefore be used for determining a non-uniform stretching of one sulcus to another that brings the curvatures of the stretched and the target sulci in best agreement.

In order to estimate the principal curvatures, $\kappa_1$ and $\kappa_2$, of the (discretized) surface $x(s, t)$, we find a least-squares quadratic patch fit around each point of the sulcal ribbon. Specifically, for each $(s, t)$ we seek the vectors $a, b, c, d, e, f$ which minimize the following error-of-fit measure:

$$
\epsilon = \frac{1}{(2M + 1)^2} \sum_{i=-M}^{i=M} \sum_{j=-M}^{j=M} |x_{i,j} - ai^2 - bj^2 - cij - di - ej - fj|^2. \quad (8)
$$

Then first and second derivatives of $x(\cdot, \cdot)$ are readily found in terms of the estimated vectors $a, b, c, d, e, f$, and the curvatures are calculated as described in Millman and Parker (1977). In order to obtain as robust a curvature estimate as possible, without sacrificing accuracy, we have implemented an adaptive window-size procedure to estimate the curvature. Specifically, we start with a small window (typically $M = N/50$, where $N + 1 \times N + 1$ is the grid size) and increase $M$ until the error of fit, $\epsilon$, reaches a predetermined value. Typically, we use $\epsilon = 0.1$ as the maximum error tolerance.

Having defined the curvatures, we then find the 2-D elastic transformation of $U$ to itself, which is driven by a force field attempting to maximize the similarity of the curvatures along two sulci. This is essentially a reparameterization of one sulcus so that points of similar geometric structure in two homologous sulci have the same parametric coordinates $(s, t)$, by locally stretching or shrinking the parametric grid based on the curvatures. The boundary conditions of this transformation fix the four corners of the sulcal ribbon, and allow the points along its four sides to slide freely under the influence of the external and the internal elastic forces. A more detailed description of this 2-D elastic deformation can be found in Davatzikos (1996a, b).

4. EXPERIMENTS

Morphological analysis

Because of the functional importance of the central sulcus and the distribution of functions along it, usually referred to as Penfield’s homunculus (Martin, 1989), it is our primary interest to quantify its morphology and to map the function along it. Therefore, in most of our preliminary experiments we focused on the central sulcus.

In our first experiment we selected a volumetric MR image from our database, acquired with the standard SPGR protocol, with voxel size $0.94 \, \text{mm} \times 0.94 \, \text{mm} \times 1 \, \text{mm}$. We then applied the deformable surface algorithm, described in Subsection 2.2, and obtained a parameterization of the outer cortical boundary. The deformable surface comprised 80,000 polygons. From this parameterization, the curvature and the inward normals on each surface were calculated. A 2-D image of the curvature of the top of the brain, which has high values at the cerebral sulci, was used to guide the manual placement of the initial configuration of an active contour consisting of 100 points. Linear interpolation was used to space equally the outlined points (see Figure 11). The active contour was initialized at these points and the algorithm was applied. A sulcal ribbon of $100 \times 100$ points generated from the resultant trajectories is shown in Figure 12. Cross-sections of the same sulcal ribbon are shown in Figure 13 superimposed on the corresponding MR images. Figures 12 and 13 show that the ribbon obtained by the algorithm lies along the medial surface of the sulcus. A grid highlighting the active contour and the trajectories of its points at regular intervals is superimposed on the sulcus in Figure 12b to illustrate the uniform spacing along the $t$-isoparametric and $s$-isoparametric curves. Figure 12c is a top view clearly showing the gyrations of the sulcus along its long axis. The parameters used here were the following: $\alpha = 0.8$, $\beta = 0.1$, $\gamma = 1$, $K = 0.2$. The total computational time of the algorithm was 22 s on a Silicon Graphics Impact workstation. We should note here that due to the complexity of the cortex, we typically apply this algorithm only on data of fairly high resolution, with a slice thickness of $1$–$1.5 \, \text{mm}$.

We then applied this procedure to four additional subjects and obtained the sulcal ribbons shown in Figure 14.
Parametric representations of the cortical sulci

Figure 11. (a) Outer cortical surface with the initial active contour superimposed. (b) The parametric domain $U$ with the minimum (principal) curvature with the initial active contour superimposed.

(including the one of the previous experiment). In order to obtain an average shape of the central sulcus in these five subjects, we applied the Procrustes fit algorithm as described in Subsection 3.1; the resulting average shape is shown in Figure 14b. The central sulcus of these five subjects appears to have a fairly consistent folding pattern. Moreover, the average sulcus reveals two prominent folds, which further suggest a consistency in the folding pattern of the central sulcus in these five subjects. One of our long-term goals is to detect and precisely quantify such stable, across individuals, prominent sulcal folds. It has been postulated in the literature (Demiau et al., 1996) that such folds might be related to specific functional regions on the cortex.

Next, we applied our algorithm to three additional sulci: pre-central, superior frontal and inferior frontal of the left hemisphere. The resulting sulci of two subjects are shown in Figure 15. Interestingly, the superior frontal sulcus appears to have a very similar folding pattern in both individuals, which is shown more clearly in Figure 16.

Validation

The performance of our algorithm was evaluated on five MR images by quantitatively comparing the sulci obtained from our algorithm with a manual tracing of the same sulci. The central sulcus was traced on the axial slices; the outlines were then stacked to a 3-D volume. Error measurements were obtained by recording the distance in millimeters between each point in the parameterized surface and its nearest point in the manually traced sulcus. The distance measure was calculated by growing a spherical neighborhood around each point in the parameterized surface until it intersected a manually traced point. The radius in millimeters of the resulting neighborhood is the error measure, which we refer to as error 1. We measured the average error, its standard deviation, and maximum error for each central sulcus of the five subjects. This error measure, however, does not reflect deep parts of the actual sulcus which were not reached by the active contour. Specifically, each point in our parameterized surface $\hat{x}(s, t)$ is close to some manually traced sulcal point. However, there may be parts of the traced sulci which are not reached by the active contour. For example, the motion of the active contour might be terminated before it reaches the root of a sulcus due to noise or partial-volume averaging in the images (see Figure 17). In this case, the ribbon extracted by our algorithm lies entirely on the sulcus. Hence, error 1 does not indicate the failure of our algorithm to trace the entire sulcus. In order to measure such errors and test the ability of our algorithm to cover a sulcus in its full extent, we also obtained a second error measure. This measure is similar to the one described above except that it measures the distance between each point in the manually traced sulcus and its nearest point in the parameterized sulcus, i.e. the neighborhood grows and is centered about each manually traced point rather than each point in the parameterized sulcus (see Figure 17). At a deep region of the sulcus that is not reached by the active contour, this error measure is high. We refer to this error as error 2.

Each sulcus was manually outlined by two different raters; the errors are shown in Tables 1 and 2. We also computed the differences between the two manual tracings by using the second error measure. The results show that the average errors fall roughly between 0.7 and 1.6 mm which are comparable with the average difference between the two raters which fall between 0.6 and 0.9 mm and have a similar variance. We conclude that manual tracing would result in only a marginal improvement, at the expense of irreproducibility, subjectivity and excessive human effort.

Figure 18a shows the average error 1 superimposed on the average central sulcus; Figure 18b shows the same
Figure 12. Central sulcus. (a) Resulting sulcal surface. (b) Sulcal surface with grid lines superimposed. The thinner and longer grid lines indicate the configuration of the active contour at various times. The thicker and shorter lines indicate the trajectory of selected points of the active contour. (c) Top view orientation of the sulcal surface illustrating gyrations along the long axis of the central sulcus. (d) Cross-sections of the MR volumetric image (left hemisphere) intersected by the sulcal surface.

error superimposed on the parametric domain $\mathcal{U}$, which was stretched in each direction proportionally to the length and depth of the sulcus. Figure 18c shows the residual variance, after the Procrustes fit, superimposed on the average sulcus;
Figure 13. Cross-sections of a resulting sulcal ribbon (central sulcus) superimposed on the corresponding MR slices.

Figure 14. (a) The resulting central sulcus of five subjects and (b) their Procrustes average.
Figure 15. (a) Four sulci (central, pre-central, superior frontal and inferior frontal of the left hemisphere, labeled a, b, c, d respectively). (b) The same four sulci of another subject.

Figure 16. The superior frontal sulci of two different subjects

Figure 17. (a) Schematic drawing of a parameterized sulcus and a manually traced sulcus. Error 1 is the radius \( r \) of the neighborhood centered at a point on the extracted surface which first intersects the manually traced surface. Note that although the parameterized surface has not reached the root of the sulcus, error 1 is small. (b) The same two surfaces now illustrating error 2, the radius of the neighborhood centered at a manually traced point which first intersects the parameterized surface. A failure of the algorithm to track the full depth of the sulcus would be reflected in error 2 as shown by the large neighborhood centered at a point along the root of the manually traced sulcus.

Figure 18d shows the same variance superimposed on the parametric domain. In interpreting the variance along the sulcus, it is necessary to look simultaneously at the average error at the same location, because variability measurements at regions of high error are dubious—high variance may be due to high error. Although somewhat difficult to picture in Figure 18, the most meaningful variance information should be considered along the regions of the sulcus which correspond to low error—seen as dark regions in Figure 18b.

Interestingly, it appears that some of the most variable regions, seen as bright regions in Figure 18d, have low error. One fairly distinct region corresponds to a fold along the center of the sulcus, indicated by an arrow, from which we have made two inferences. First, the low error indicates that our algorithm has been successful in following a seemingly difficult, high-curvature region of the sulcus. Secondly, the homunculus would somatotopically map the facial regions of the body approximately to what we have found to be a highly
variable region of the central sulcus. One might expect such shape variability because of the high variability in facial expressions, which are partly controlled by that region of the primary motor cortex, as compared to the feet, hands, etc. (Luria, 1966). The statistical power of this study, however, is insignificant. The results need to be replicated on a larger sample.

Thompson et al. (1996) have performed similar experiments on sulcal variability. The main differences between our approach and the approach in Thompson et al. (1996) are the automated algorithm for parameterizing the sulci, the use of the Procrustes fit prior to averaging and the measurement of the variance of the component perpendicular to the tangent of the sulcus, which as explained earlier is expected to be subject to a smaller error.

**Effect of the parameters**

We next measured the dependence of the performance on the selection of the four key parameters of the active contour formulation: $\alpha$, $\beta$, $\gamma$ and $K$. We measured the two errors described above; the results are shown in Figure 19.

Figure 19a shows a rather sharp increase in error 2, while error 1 is stable for several orders of magnitude. Large values of $\alpha$ dampen the motion of the active contour, resulting in its early termination. Error 1 remains small, since the active contour does not deviate from the medial surface of the sulcus, it merely stops prematurely. Figure 19b shows a rather stable behavior for a range of $\beta$ over several orders of magnitude. Large $\beta$ result in large inertial forces; in its attempt to maintain its velocity, the active contour then deviates from the medial surface of the sulcus and its motion is terminated. Figure 19c shows a stable behavior over a range of more than two orders of magnitude. Large $\gamma$ result in overshooting of the active contour; the active contour then goes too deep into the sulcus and error 1 increases. Finally, as Figure 19d shows, if the active contour internal forces become too large, they dominate over the forces driving it toward the root of the sulcus, resulting in poor solutions.

**Sulcal interruptions**

In this experiment we demonstrate the performance of our active contour algorithm in the presence of interruptions. Specifically, we applied our algorithm to the pre-central sulcus—one of the frequently interrupted sulci with a rather complex shape. The resulting ribbon is shown in Figure 20a. A major total interruption is indicated by the arrow a; the trajectories of the active contour points in the vicinity of that interruption terminated in the very beginning of the algorithm, causing the active contour to wrap around the underlying gyrus. Two smaller interruptions are indicated by the b arrows.

The first fundamental form coefficient $G$ was calculated for this ribbon and it is displayed in color overlayed on the sulcus in Figure 20b; red implies a large $G$ and blue implies a small $G$, the pure blue corresponding to $G = 0$. The same coefficient is shown overlayed on the parametric domain $U$ in Figure 20c, which for display purposes has been stretched horizontally according to the average length–depth ratio of the sulcus; here bright corresponds to red (large $G$) and dark corresponds to blue (small $G$). From this figure we clearly see that the first fundamental form coefficient reflects the location and extent of the depth of the sulcus and therefore provides a quantitative way of representing its interruptions.

**Functional mapping**

To illustrate the application of our methodology to functional mapping, we have mapped fMRI data from a left-hand finger-tapping experiment, to the central sulcus of the right hemisphere. The experiment was a complex finger tapping, i.e. a sequential tapping of all fingers in one direction followed by a sequential tapping in the opposite direction. This kind of

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Figure 19. Error plots for various parameters of the active contour formulation.

experiment was expected to activate a fairly wide area along the primary motor cortex, anteriorly to the central sulcus.

fMR images with a slice thickness of 7 mm were extracted and processed using a standard time-series correlation analysis. Points with significant correlation coefficient were ‘labeled’ as activated points. MR SPGR images of the same subject were also obtained, with slice thickness equal to 1.5 mm. The central sulcus was then extracted from the anatomical images using our algorithm and then each activated point in the fMRI was mapped to the nearest point on the central sulcus, resulting in the activation maps shown in Figure 21. It appears that the activated regions along the central sulcus fall approximately where we would expect the somatotopic projection of the hand to be. Clearly, a relatively wide area along the primary motor and sensory cortices was activated in this experiment, which is typical in fMR images. Studying the distribution of the function along the cortex on either side of the central sulcus and quantifying its variability across individuals is possible using the sulcal mapping method described herein.

3-D warping
In this experiment we tested the utility of the parametric representations of the sulcal ribbons in obtaining a more accurate non-rigid registration in the cortical region. In particular, we warped a volumetric MR image of one individual to that of another, using the outer cortical surface, the ventricular surface and four sulci (central, pre-central, superior
Figure 18. (a) Average error 1 color-mapped along the average central sulcus (blue indicates the lowest error and red highest error). (b) Average error 1 superimposed on the parametric domain. (c) The residual variance along the central sulcus, after the Procrustes fit (blue indicates the lowest variability and red the highest variability). (d) The same variance superimposed on the parametric domain.

Figure 20. (a) An interrupted pre-central sulcus; arrow a indicates the location of a total interruption and arrows b indicate smaller interruptions. The grid lines are superimposed, illustrating the deformation of the active contour as it wraps around the interruptions. (b) The first fundamental form in color scale, superimposed on the sulcus. Red indicates a large $G$ and blue indicates a small $G$. (c) The first fundamental form in gray scale, superimposed on the parametric domain $U$. Bright corresponds to large $G$ and dark corresponds to small $G$. 
Figure 21. (a) fMRI finger-tapping activation mapped along the central sulcus (activation shown as dark spots). (b) 2-D flattened map of the sulcus (activation shown as white spots).

Figure 22. 3-D elastic warping using the outer cortical surface and four sulci of the left hemisphere (right-hand side in the images) as landmarks. (a) A cross-section of the warped image. (b) The same cross-section overlayed on the cortical outline of the target image. (c) The corresponding cross-section of the target image. The arrows a and b highlight the sulci where a better registration was achieved. A poorer registration, in which the four sulci were not used as features, is seen along the analogous sulci, c and d on the other hemisphere.

Figure 23. One of the principal curvatures along the central sulcus, the parameterization of which was obtained from an MR image: (a) sulcus 1, (b) stretched (reparameterized) sulcus 1, (c) sulcus 2. Both curvatures are defined in the parametric domain \( U \). A grid is overlayed for comparison purposes. Each grid line is mapped to a 3-D curve along the corresponding sulcus; the distance between these curves in real 3-D space is \( \sim 18 \) mm.
frontal and inferior frontal, only of the left hemisphere) as features (see Figure 15). A uniform stretching was applied to corresponding sulci to map them to each other. A cross-section from the warped image is shown in Figure 22a, and in Figure 22b overlayed on the cortical outline of the target image, which is shown in Figure 22c.

It is apparent from these images that better registration was achieved at the left hemisphere (right-hand side in the images, according to the radiology convention), which is the hemisphere for which the four sulci were used as features. Most notable is the registration around the central and superior frontal sulci, indicated by the arrows.

**Nonuniform stretching of the sulci**

In our final experiment we demonstrate the procedure for elastically reparameterizing the sulci, based on their principal curvatures. Figure 23a and c displays the maximum curvature along the central sulcus (extracted using the described algorithm) of two brains, defined in the parametric domain $U$. The two curvature peaks correspond to the two prominent folds of the central sulcus. Figure 23b shows the curvature of the reparameterized sulcus of Figure 23a, which has been stretched along the vertical axis in the image (which corresponds to the length of the sulcus), conforming to that in Figure 23c. For comparison purposes, a grid of lines, spaced at $\sim 18$ mm in real space, is shown overlayed on the images. Better agreement between the curvatures in Figures 23b and c is apparent, compared with that between Figure 23a and c. We note that this procedure does not change the shape of the sulcus corresponding to Figure 23a, since our reparameterization preserves the extrinsic geometry of the surface.

5. DISCUSSION

We have developed a quantitative methodology for representing the shape of the sulci, utilizing particular characteristics of the cortical folds. A force field guides an active contour along the medial surfaces of a sulcus, thereby parameterizing the sulcal ‘ribbons’. The application of this technique was demonstrated in quantitative morphological analysis, functional mapping and spatial normalization and registration of brain images.

A key issue in our method is that a parameterized representation of the cortical folds is obtained automatically. The limitation of previous methods for medial axis finding is the difficulty in parameterizing the result. Traditional axis-width descriptors as described in Blum (1967), Blum and Nagel (1978), Morse et al. (1994) and Pizer et al. (1994) often generate noisy medial axes or more importantly disconnected axes, making parameterization and subsequent shape analysis very difficult. Moreover, axes in the whole image are extracted, most of which do not correspond the sulci and therefore must be manually removed at a post-processing step.

Several extensions of this basic approach are possible. In particular, a current limitation of our algorithm is the requirement for the manual initialization of the active contour. The automation of this procedure is a very difficult task, primarily because of the complexity of the cortical structure. Specifically, although sulcal edges can be identified from the peaks of the absolute value of the minimum curvature of the outer cortical surface (Thirion et al., 1992; Davatzikos and Bryan, 1995; Subsol et al., 1996), the differentiation between the sulci eventually requires a higher-level model. Our current and future work in this direction focuses on the use of prior probability distributions which reflect our expectation for finding a particular sulcal edge at a given coordinate $(u, v)$ of the outer cortical surface $b(u, v)$. These priors, together with geometric properties of the outer cortex, such as curvatures, can potentially lead to the automatic identification of specific sulci and the initial placement of the active contour along them.

A key problem in the analysis of the morphology of the sulci is their variability in topology. Specifically, most sulci have interruptions that, depending on their depth, can be either total or partial. In our framework we have dealt with this difficulty by allowing the active contour to ‘wrap’ around an interruption, thereby always obtaining a map from a standard domain, the unit square $U$, to the whole sulcal ribbon, whether or not the latter is interrupted. This approach effectively treats the sulcus as a continuous structure, which is partially or totally ‘squashed’ at certain regions, due to the growth of underlying gyri, and conveniently allows us to quantify sulcal interruptions through the first fundamental form coefficients, as was shown in Section 4, Figure 20. However, an alternative method to treat sulcal interruptions is to assume that the growth of a gyrus eliminates, instead of pushing upwards, part of a sulcus. In this case, our active contour could be allowed to split whenever it encounters an interruption, resulting in a number of ribbons, that are partly joint and partly split. Although this is clearly an alternative way worth investigating, it does not readily lend itself to quantitative analysis of the sulcal interruptions, as the approach presented herein does.

Several issues related to the three applications presented in Section 3 need to be addressed. Specifically, extensive analysis of the shape of the sulci needs to be carried out, which will reveal those geometric characteristics of the sulcal ribs that consistently appear across individuals. Several major sulci are known to have such consistent shape features, related to the associated cortical convolutions, although most secondary and tertiary sulci do not. Features that are consistently found across individuals and that can be reliably extracted from MR images using our approach can then be used
to establish sulcus-to-sulcus maps. Such more precise maps will reduce the problem of overly smooth averages shown in Figure 14b, which is a result of the misregistration of the sulcal folds.

Moreover, precise sulcus-to-sulcus maps will result in more precise 3-D non-rigid registration transformations, which are important in the cross-subject analysis of functional activation images (Friston et al., 1995). However, the exact relationship of geometric features, which current registration methods are using, with the underlying functional organization of the brain must be extensively studied. Current and future directions of research in our laboratory aims at addressing these issues.

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