ABSTRACT
This paper presents a kernel-based method of correspondence detection in diffusion tensor images (DTI), a key step towards their deformable registration. The proposed method is driven by a few focus points chosen in white matter, characterized by a unique morphological signature which incorporates the anisotropy, orientation and the anatomic context by using “oriented” Gabor filters and several candidate matches for each focal point, defined using tensorial similarity of these orientation specific signatures. The final focal point correspondences are defined via minimization of a function that seeks to satisfy three criteria: sparsity, which enforces unique correspondence, tensorial similarity of Gabor features, and smoothness, and are obtained by solving a constrained non-linear optimization problem with inequality bound constraints, using an optimization solver that employs primal-dual interior point algorithms and which ensures global convergence. The solution of the optimization problem produces the best correspondences for the focal points and uses these correspondences to obtain the optimal kernelized interpolation parameters for non-focal points. Experimental results on human brain data in which datasets with tumor are matched with normal brains, demonstrates the ability of the method in determining very good correspondences in the white matter, and its applicability to datasets with large mass effect as in tumors.

1. INTRODUCTION
The clinical importance of using diffusion tensor imaging (DTI) [1] in studying white matter anatomy especially in large population studies, has fueled the need for a tensor-based deformable registration of DTI. Existing DTI registration routines estimate the non-linear elastic warping by minimizing locally optimized scalar (fractional anisotropy [2]) or tensor [3] similarity measures between the template and the subject image followed by the reorientation of the tensors [2] or register DT images using a multichannel framework [4] combining several scalar measures computed from DT images. These methods use image intensity based local information to drive the registration, which are not rich enough morphological signatures to characterize the tensors with their inherent orientation information. The registration itself is modeled as an unconstrained non-linear optimization problem which is usually solved using the steepest descent method, that only ensures a linear rate of convergence and therefore could be arbitrarily slow indicating that a more efficient optimization [5] will yield a better registration.

Since DTI provides an excellent characterization of white matter, but is less reliable in gray matter, greater success in DTI registration can be achieved through a feature-based approach in which the registration is driven by a few automatically chosen focus points in the white matter, characterized by their morphological signatures. While these focus points can be chosen manually, in the interest of reliability and speed in clinical studies, an automated method of correspondence detection is greatly desirable. We propose an automated correspondence detection framework based on similarity of tensors characterized by a rich and distinctive morphological signature using oriented Gabor filters which provide an anatomical, spatial and orientational context for each tensor. We model automated correspondence detection as a non-linear constrained optimization problem with bound constraints based on kernelized combination of a relatively small number of focal points for which the morphological signatures have been evaluated. Then using the tensor-based similarity measure defined on the morphological signatures several candidate matches are identified for each of these focal points, thereby jointly constituting a “fuzzy” correspondence. The automated correspondence detection is solved by employing highly efficient largely scalable interior point based solvers. The solution of this optimization problem yields correspondences at the focal points, and interpolation parameters to determine correspondences at the non-focal points also chosen in the white matter, or with sufficient white matter in the vicinity to provide a white matter context. These correspondences, which reinforce the white matter regions, can then be incorporated into any registration algorithm [6] which provides a good registration of gray matter, to obtain a spatial normalization of DTI.

2. FRAMEWORK FOR CORRESPONDENCE DETECTION
We uniquely characterize tensors through a rich and distinctive morphological signature defined using “oriented” Gabor filters. Similarity of these signatures is based on tensorial measures. Details of the method can be found in [7]. Us-
ing these morphological signatures we have developed an au-
tomated kernel-based correspondence detection scheme for DTI.

We solve correspondences detection using the function \( f \) which deforms the subject \( S \) to the template \( T \). \( f \) induces a displacement field \( d = (dx, dy, dz) \) such that if \((X_i, Y_i)\) is a pair of corresponding points such that \( X_i \in S \) and \( Y_i \in T \), then \( Y_i = f(X_i) = X_i + d(X_i) \). The displacement \( d \) is in turn defined in terms of correspondences of a small number of focal points chosen in the white matter, that have distinct morphological Gabor signatures. Several candidate matches are identified initially for each focal point which form a fuzzy correspondence for this focal point. This requires that spar-
sity be incorporated into the framework, so that only one of the possible matches is finally chosen for each focal point. Unique correspondences are determined as a solution of a non-linear minimization problem modeled as a combination of interpolation on focal points, sparsity of correspondences of the focal points and smoothness of the transformation. Opti-
mal interpolation parameters produced can be used to obtain matches at the non-focal points.

**Formulation for Interpolation error:** We adopt a kernelized representation of deformation. By the Representer theorem from the theory of Recursive Kernel Hilbert spaces (RKHS), \( f \) as an interpolant is obtained by minimizing
\[
\frac{1}{\pi} \sum_{i=1}^{n} G(f(X_i), Y_i) + \lambda |f|^2,
\]
where \( G(.,.) \) is some cost function for the match between \( X_i \) and \( Y_i \); and on solution, has the form \( f(.) = \sum_{i=1}^{n} a_i K(X_i, .) \), where \( X_i \) is the members of an index set and \( K(.,.) \) be a positive definite function on this index set equipped with inner product \( \langle < K_i, K_i > = K(s, t) \). By suitably restricting \( f \), we can obtain a similar form for \( d \). That is, we model the displacement function \( d \) as a linear combination of a set of kernel evaluations at the focal points \( \{X_i\}_{i=1}^{M} \) spread out over an irregular grid on the image such that \( d(X_k) = \sum_{i=1}^{M} w_i K(X_k, X_i) \), \( 1 < k < M \) where \( w_i \in \mathbb{R}^3 \) and \( K : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R} \) is a positive definite kernel which guarantees that the system is solvable to obtain the interpolation parameters \( w_i \), which will determine the correspondences for all non-focal points. The focal points \( X_i \) are chosen to drive the registration and model the deformation field. These are automatically selected voxels with distinctive anisotropy and orientation information and can also be points on fiber tracts and their intersections.

If \( Y_i, i = 1, \cdots, M \) are the exact point correspondences for \( X_i, i = 1, \cdots, M \) then, \( d(X_i) = Y_i - X_i \). However, we relax the uniqueness of correspondence to impose flexibilitiy into the system, such that to each focal point \( X_i \), we associate a set \( S_i \) of candidate matches, represented by \( Y_{ij}, j = 1, \cdots, N_i \) such that \( \sum_{j=1}^{N_i} N_i = N \) is the total number of candidates. These candidates are chosen on the basis of tensor similarity measures defined on the morphological signatures of the tensors at these focal points and their correspondences. With each candidate \( Y_{ij} \) we associate a scalar \( c_{ij} \) which indi-
cates its contribution towards the correspondence. In the final solution, ideally all \( c_{ij} \)’s with respect to each \( i \) would go to zero, except the one which is the actual correspondence. We thereby force the actual correspondences to lie in the space of candidate matches. The interpolation error can now be expressed as:
\[
\text{Interpolation Error} = \sum_{i=1}^{M} e_i e_i^T
\]
where \( e_i = \sum_{j=1}^{N} c_{ij} Y_{ij} - X_i - \sum_{k=1}^{M} w_k K(X_k, X_i) \).

The choice of the kernel is crucial for the overall smooth-
ness or the locality of the warping transformation and we use a kernel with compact support [8], which has a well defined region of influence, and far away points do not affect correspondences of each other, making the process more efficient. These are more efficient than Gaussian kernels which can have a large region of influence and thin plate spline kernels which increase with the increasing distance from the focal point and have a global effect. In addition, we found that thin plate splines dominated the sparsity condition and hindered exact correspondences.

**Sparsity formulation:** In order to achieve unique correspon-
dence for the focal points, we impose a sparsity structure on the \( c_{ij} \)’s, that is, we favor solutions for which all but one of the \( c_{ij} \) are zero. Candidates correspondences are chosen based on the morphological signature similarity \( s_{ij} \) between each can-
didate \( Y_{ij} \) with its focal point \( X_i \), with higher similarity to have a greater contribution towards the final solution.

\[
\text{Sparsity term} = \sum_{i=1}^{M} \sum_{j=1}^{N_i} |(1 - s_{ij}) c_{ij}|^p, \quad 0 < p < 1
\]
such that \( \sum_{j=1}^{N} c_{ij} = 1, i = 1, \cdots, M \), which warrants that not all \( c_{ij} \)’s become zero simultaneously. During minimiza-
tion, the p-norm like diversity measure [9] tries to reduce the number of non-zero \( c_{ij} \) and at the same time tries to pick those with minimum value of \( (1 - s_{ij}) \) and hence maximum value of the similarity \( s_{ij} \).

**Smoothing Criterion:** We impose smoothness on the deforma-
tion field through the coefficients of the kernel and since these are explicitly provided by the the vectors \( w_i \), we define the smoothness in terms of inner product on these.

**The Optimization Problem:** We can now formulate the optimization problem in terms of the interpolation parameters \( w_i \) and the sparsity parameters \( c_{ij} \)
\[
\min_{c_{ij}, w_i} \lambda \text{(Interpolation Error)} + \mu \text{(Sparsity term)} + \delta \text{(Smoothness term)}
\]
with the constraints \( \sum_{j=1}^{N} c_{ij} = 1, \forall i = 1, \cdots, M \) and \( 0 \leq c_{ij} \leq 1, \quad j = 1, \cdots, N_i, i = 1, \cdots, M \), \( \Sigma_{i=1}^{M} N_i = N \). While
the first term in the objective function balances the error between the exact match and the match obtained from interpolation, the sparsity term controls the correspondence detection and the tensor similarity in a sparsity formulation and the smoothness term controls the smoothness of the deformation field produced. The solution of the optimization problem would result in sparsity being imposed on the correspondences, producing unique matches for the focal points and interpolation parameters for the non-focal points.

The system depends on several parameters: $\sigma$, the kernel parameter which determines the level of smoothness, $p$, the sparsity parameter, $\lambda$, the parameter associated with the term for registration errors, $\mu$, the parameter associated with the sparsity term and $\delta$, the parameter associated with the smoothness error term in the objective function. We define an additional parameter called the sparsity index, which is the difference between the $c_{ij}$ values prior and post optimization. Higher this index, more is the sparsity. We choose the set of best correspondences to be the set with the lowest registration error and smoothness error and the highest sparsity index. These parameters are empirically chosen. The choice of $\lambda$ and $\delta$ determine the balance between the interpolation error and smoothness of the deformation fields. Comparable values of $\lambda$ and $\delta$ produce low interpolation error, smoothness and high sparsity.

The problem comprises of a non-linear objective function with $M$ linear equality constraints (equal to the number of focal points initially chosen), and $N$ bound constraints on the sparsity variables $c_{ij}$’s (equal to the total number of candidates for all the tensors). When focal points are in $\mathbb{R}^3$, we solve for $3M + N$ variables, which increase with every additional focal point. In order to address problems of this size, we need to choose an appropriate solver which will provide a practically feasible solution. Motivated by the scalability of the interior point approach over active set strategies [5], we use IPOPT [10], a nonlinear programming solver employing the primal-dual interior point method in conjunction with filter methods that ensure global convergence, that is, our solution will reach a local optima irrespective of the starting point supplied, although the convergence may be dependent on the starting point. We compute the gradient analytically and use quasi-Newton updating of the Hessian. We stop when the error of constraint violation of the optimality conditions $\|E\| = \|E\|_0 \times 10^{-8}$, where $\|E\|_0$ is the error at the initial iterate.

Summarizing the automated correspondence detection framework, we rigidly align the subject to the template and identify the focal points $X_i$’s on the template and compute the morphological signature for these focal points. These focal points are about 500 points placed on an irregular grid covering the prominent fiber tracts and regions of high fractional anisotropy. Using the tensor-based similarity measure on the morphological signature for all points on the subject, we identify almost 10 possible matches $Y_{ij}$ with similarity $s_{ij}$ for each of the tensors chosen as focal points. Then the objective function is set up, with the interpolation term, the sparsity term and the smoothness term, together with the linear and bound constraints and solved as a non-linear constrained minimization problem using an interior point based solver (in our case IPOPT [10]) and obtain the deformation field. The correspondences so generated can be incorporated into any elastic warping method [6] to obtain complete warping.

3. EXPERIMENTS AND DISCUSSION

The correspondence detection framework is tested on human brain data. In addition, rigorous experiments are conducted on simulated and real data to analyze the pattern of variation of parameters for our formulation.

**Variation of parameters** Fig.1 (a) shows the template image with some focal points identified on it using red dots. Fig. 1 (b) shows the candidate positions for different values of the sparsity parameter $p$ and the yellow dots show the best correspondence in terms of their distance from the actual correspondences of these focal points marked on the subject by an expert, and which serve as the gold standard. We deliberately introduced error prone matches in the form of low

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Fig. 1. (a) focal points on the template (b) position of correspondences obtained after optimization with different values of $p$, yellow dots represent the best correspondence, red and green dots represent sub-optimal correspondences with alternate $p$ values

Fig. 2. Variation of smoothness error, sparsity index and interpolation error with change in (a) $\sigma$ (b) $p$
anisotropy points. However the system converged to the true correspondence due to the interpolation and smoothness constraints, which prevented the point from straying too far from its correspondence.

In Fig. 2, we show the effect of varying $p$ and $\sigma$ on the interpolation error, smoothness and the sparsity index. It can be seen that with low values of $\sigma$, there is a very high smoothness error, this gets reduced with increasing $\sigma$ till 40. However at greater values, the smoothness error starts increasing. Although the interpolation error increases with increasing $\sigma$, it plateaus after $\sigma \approx 50$. Therefore the ideal range of variation of $\sigma$ is between 35 and 60. In Fig. 2 (b), we study the variation with respect to $p$. Acceptable range of $p$ is between 0.7 and 0.99. Lowering $p$ further decreases the sparsity and increases the smoothness and interpolation error. The increase in interpolation error can be explained by the fact that a low sparsity value of $p$ causes the optimization to converge to a sparse solution without permitting the interpolation and smoothness terms to come into play.

Finally, we demonstrate correspondence detection on images with tumor which cause a large deformation owing to mass effect. In Fig. 3 (a), we identify several white matter focal points on a healthy brain. Fig. 3 (b), shows a slice with tumor, which has deflected the corpus callosum, and has caused a thinning of the tracts around the tumor. It can be seen that correspondences are correctly identified around the tumor, as well as points on the internal and external capsule chosen far from the mass effect of the tumor. Thus the correspondence detection framework is able to identify point correspondences even in the presence of large mass effect, based on the white matter context provided by the Gabor morphological signatures. This demonstrates that our correspondence detection framework can therefore be integrated into a landmark-based registration framework, which can guide registration in areas with large mass effect, as around tumors.

4. CONCLUSION AND FUTURE WORK

We have provided a new kernel-based method for automated correspondence detection in DTI, in which tensors are characterized by oriented Gabor filters. This is then extended to a framework for the spatial normalization of DTI data in which these automatically detected correspondences provide the desired white matter context and can be used as landmarks to guide registration, in areas of large deformation, as around tumors, where traditional spatial normalization techniques based on intensity [6] do not perform optimally. The problem is modeled as a constrained non-linear optimization problem which jointly solves the interpolation and correspondence detection problem in an interior point framework. The method is general and applicable to registration of all volumetric data which can be characterized by attribute vectors. We are currently addressing the issues of DTI filtering and interpolation using Gabor filter based morphological signature of tensors. Further investigation is necessary to determine appropriate strategy for focal point placement and distribution in the image volume and study the applicability of other available solvers to our class of problems.

5. REFERENCES