MATCHING OF DIFFUSION TENSOR IMAGES USING GABOR FEATURES

Ragini Verma and Christos Davatzikos

Section of Biomedical Image Analysis, Department of Radiology, University of Pennsylvania
3600 Market Street, Suite 380, Philadelphia, PA 19104
{rverma,christos}@rad.upenn.edu

ABSTRACT

This paper presents a novel method for feature-based matching of diffusion tensor images using the complete tensor information available at each voxel rather than limited scalar parameters such as the fractional anisotropy. In our method, we characterize each voxel by a rich rotationally invariant feature vector defined using Gabor filters. In order to obtain these features, the Gabor filters are evaluated at multiple scales and frequencies and are oriented along the dominant direction of the tensors in a neighborhood around the voxel under consideration. The feature is able to obtain a fine to coarse description of each voxel and fully accounts for the highly oriented nature of the tensor data. The proposed matching paradigm based on these Gabor features has been tested on simulated and real images and produces good correspondences.

1. INTRODUCTION

In this paper, we present a novel method of matching two Diffusion Tensor Images (DTIs) by detecting correspondences using Gabor features. The tensor information in each voxel is characterized through an attribute vector (AV). These AVs are computed by using three dimensional Gabor filters at multiple scales and multiple frequency levels but oriented along the dominant direction of the neighborhood of the tensor at the voxel under consideration and are called Gabor features.

In DTI, the measurement acquired at each voxel describes the local water diffusivity of the material being imaged [1]. This directional diffusivity signifies the connectivity between anatomical structures, and corresponds to neuronal fiber connectivity between the various regions of the brain. This is important as brain connectivity is crucial to investigations in brain development, aging and disease progress. For such applications, it is important to develop techniques for DTI analysis which include spatial normalization (registration) and subsequent tensor reorientation. The existing methods of analyzing DTIs [2] typically convert the multi-dimensional tensor information at each voxel to a single dimensional entity using some anisotropy measure provided by DTI data like fractional anisotropy, relative anisotropy and volume ratio [3], causing a loss of information. Although these methods work for intra-patient registration, there is a large variability in the tensor maps across patients. Also the data is noisy and prone to distortion. In such cases, a simple intensity based analysis will not suffice as all the orientation information available around each voxel needs to be taken into account. This requires a voxel-based method for matching and registration.

In order to match two images, correspondences need to be established between them. It is important to characterize each voxel using distinctive attributes (based on intensity, orientation etc.) A rich combination of attributes can differentiate two anatomically different points although they appear indistinguishable using simpler measurements like image intensity or edge strength. We provide such an AV in our Gabor Features.

Classically two dimensional Gabor filters have been used to design features for 2D texture analysis [4] and in medical imaging [5, 6]. We chose Gabor filters for feature definition for several reasons: 1) DTIs represent water diffusivity in each voxel using a tensor and have a highly directional structure, which is well characterized by Gabor filters. 2) Each Gabor filter includes a Gaussian component which helps smooth the DTI data around each voxel under consideration. 3) By design, a Gabor filter bank includes filters at different scales which facilitates understanding of voxel similarity at different scales. Also it paves the way for the design of a hierarchical strategy for the computation of voxel correspondences between two DTIs.

We use the Gabor AVs to detect correspondences between two DTI images, based on an Euclidean similarity measure. Our novel voxel-based matching paradigm using Gabor features has been tested on real and simulated data and it produces good correspondences.

2. DESIGNING GABOR FEATURES

In this section we describe the construction of attribute vectors based on Gabor filters.
2.1. The mathematics of DTI data

Each voxel in a DT image is a second order symmetric tensor $D$ represented by a $3 \times 3$ symmetric matrix as $D = (D_{ij})$, $i = 1, 2, 3$, $j = 1, 2, 3$ and $D_{ij} = D_{ji}$. This can be written as $D = \lambda_1 e_1 e_1^T + \lambda_2 e_2 e_2^T + \lambda_3 e_3 e_3^T$ where $\lambda_i$, $i = 1, 2, 3$ are the eigenvalues in decreasing order and $e_i$, $i = 1, 2, 3$ are the corresponding eigen vectors. $e_1$ is the principal eigen vector of the tensor and is the direction of maximum diffusivity.

2.2. Dominant Orientation of a Neighborhood

Let $D$ be the diffusion tensor at the voxel $(x, y, z)$. Let $N_D$ be a neighborhood around the pixel (cf. Fig. 1). The arrows at each grid point represents the principal eigen vector of the tensor at that point. The dominant orientation of this neighborhood can be found by doing a Principal Components analysis (PCA) [7] on the principal eigen vectors in this neighborhood and choosing the first principal component as the dominant orientation. This is shown by a dotted arrow $K$ in Fig 1.

2.3. Attribute vectors computation

Gabor functions are a complex exponential modulated by a Gaussian envelope. A three-dimensional Gabor mother function is defined as

$$G(x, y, z) = S(x, y, z)E(x, y, z)$$

where $S(x, y, z) = \frac{1}{(2\pi)^{1/2} \sigma_x \sigma_y \sigma_z} e^{-\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2} + \frac{z^2}{2\sigma_z^2}}$ is the Gaussian component and $E(x, y, z) = e^{j2\pi W(K^T x y z)}$ is the complex exponential. Here $\sigma_x, \sigma_y, \sigma_z$ characterize the spatial extent and bandwidth of $G$ along the respective axes and $W$ is the shifting frequency parameter in the frequency domain. $K$ specifies the orientation of the filter. In literature, Gabor filter banks have been generated at multiple orientations and scales by dilation and rotation of the mother function [4, 5, 8, 6].

We define a new modified “oriented” filter bank in which we fix the orientation of the filter along the dominant orientation of the neighborhood (cf. section 2.2). The filter is then evaluated at multiple scales (neighborhood sizes) and frequencies. The lower frequencies and scales characterize local features and higher frequencies and scales depict global features. Thus we are able to obtain a coarse to fine representation around a voxel.

Let $S$ denote the number of scales (neighborhood sizes) and $F$ the total number of frequencies. Thus the filter bank is obtained as

$$G_{s, f}(x, y, z) = G_s(W_f(x, y, z))$$

where $s = 1 \cdots S$ and $f = 1 \cdots F$. We evaluate these filters at 4 scales and 4 frequencies. The filter response on a neighborhood $N_D$ is given by

$$R_{s, f} = G_{s, f}(D(x, y, z))$$

where $\ast$ denotes convolution. $R_{s, f}$ is a 6 dimensional weighted combination of tensors where the weights are the Gabor filter coefficients. We then compute the mean of the magnitude of the transform coefficients

$$\bar{D}_{s, f} = \int \int \int_{N_D} |R_{s, f}(D(x, y, z))| \, dx \, dy \, dz$$

These 16 means (for 4 scales and 4 frequencies) are used to define the attribute vector which will distinguish one voxel from another. We now define the attribute vector for the voxel $(x, y, z)$ as:

$$AV(x, y, z) = \{ F \theta \bar{D}_{1,1}, \cdots, \bar{D}_{4,4} \}$$

Here $F$ is the fractional anisotropy (FA) of the tensor at the voxel $(x, y, z)$ and $\theta$ is the relative orientation of the principal eigen vector $\epsilon_D$ of $D(x, y, z)$ with the dominant orientation $K$ (see Fig. 1). This is the Gabor feature for this voxel.

Selection of parameters : An important aspect in the design of Gabor wavelets, is the choice of parameters $\sigma_x, \sigma_y, \sigma_z$ and $W$. Depending on this choice, the real part of the Gabor filters act as smoothing filters and the imaginary part as an edge detection filter. We have experimented with two sets of parameters. In the first set, we set the $\sigma$ values as the length of the sides of the neighborhood on which the filter is computed. Secondly, we have adapted the parameters of [4] to define $\sigma_x = \frac{1}{2\pi \sigma_w}, \sigma_y = \frac{1}{2\pi \sigma_w}$ and $\sigma_z = \sigma_x / 3$ where $\sigma_w = \frac{(s-1)U_h}{(s+1)2 \ln 2}$. $\sigma_w$ is the spread of the Gaussian envelope along the axes. $U_h$ is the number of frequencies and $S$ is the total number of scales, $f$ and $s$ are individual frequency and scales. We compute Gabor wavelets at 4 frequencies and 4 scales.

Rotational Invariance : It can be proved mathematically that these filters evaluated at a particular orientation are rotationally invariant. However to consolidate the orientation further, we reorient the tensors in the neighborhood of the voxel under consideration, in the direction of the dominant orientation, thereby reorienting the mean of the tensors in the direction of maximum variability.

2.4. Similarity criterion

Let $AV_i$ and $AV_j$ be the attribute vectors corresponding to two voxels $P_i$ and $P_j$ respectively. Then the distance
between the two is given by

\[ \text{dist}(AV_i, AV_j) = |\theta_i - \theta_j| + \sqrt{\sum_{k=1}^{16} (\tilde{D}_{ki} - \tilde{D}_{kj})^2} \] (2)

Before computing the distances, these features are normalized by using the variance of the attribute vectors computed for high anisotropy points in the image. The FA values are used for point selection as will be explained in section 3.2.

3. MATCHING PARADIGM

In this section we give the algorithmic framework for obtaining attribute vectors. Using this, we design the paradigm for establishing correspondences and matching. Let \( I_1 \) and \( I_2 \) be the two images to be matched (see Fig. 2).

![Fig. 2. Attribute vector computation and matching between two images \( I_1 \) and \( I_2 \)](image)

3.1. Attribute vector computation

1. Pick a point \( P(x, y, z) \) in the image \( I_1 \). We will be computing the feature vector for this point using 4 scales (neighborhoods) and 4 frequency values. Let \( e_i, i = 1, \cdots, 3 \) denote the three eigen vectors of the tensor \( D \) at \( P \).
2. For the given scale \( s \), define a neighborhood \( N_P \) around \( P \). This defines the tensor patch under consideration.
3. Perform PCA on the principal eigen vectors of all the tensors in the neighborhood, and obtain the dominant orientation \( K \) (see section 2.2 for details).
4. Compute the angle between the \( K \) and \( e_1 \), the principal eigen vector of \( D \).
5. Reorient the tensors towards the dominant direction.
6. Compute the Gabor filter at the dominant orientation and 4 frequencies, \( f = 1, \cdots, 4 \) and apply to the tensor patch. The responses are \( R_{s,f} \).
7. Divide the scale by 2 and repeat steps 2,3 and 4. Repeat process 4 times, thereby obtaining 16 filter responses \( R_{s,f} \), \( s = 1, \cdots, 4, f = 1, \cdots, 4 \).
   The attribute vector is as shown in Eq. 1.

Implementation details:

1. In images of sizes \( 256 \times 256 \times 50 \), the neighborhoods are chosen to be of sizes \( 40 \times 40 \times 12, 20 \times 20 \times 6, 10 \times 10 \times 3 \) and \( 4 \times 4 \times 1 \) (these may be altered proportionately, depending on the image size). In order to reduce the implementation complexity and time, we pick points is the larger neighborhoods sparsely. That is, if the neighborhood is of size 40, we pick points which are 4 pixels apart, in neighborhoods of size 20, the points are 2 pixel apart and in neighborhoods of size 10 and 4, all points are used for computation. This can be seen for a single slice in Fig 2.

2. The attribute vectors are normalized using the variance of the attribute vectors computed at points of high anisotropy.

The above process is used to compute attribute vectors for all points (with non-zero fractional anisotropy) for both the images. These are then stored in \( GF_1 \) and \( GF_2 \), for use in the matching process, which we explain next.

3.2. Framework for Correspondence Detection

Let \( P \in I_1 \) be the point under consideration. The aim of the matching process is to find the points \( P' \) which corresponds to \( P \) anatomically. The procedure we follow is:

1. Let \( AV_1(P) \) be attribute vector of \( P \) obtained from \( GF_1 \). \( F(P) \) is the FA value at \( P \).
2. Find the point \( \hat{P} \in I_2 \) to which \( P \) would have been mapped when \( I_1 \) is globally affine registered to \( I_2 \) [9]. \( \hat{P} \) is the initial search point.
3. Pick a \( 30 \times 30 \times 10 \) neighborhood around \( \hat{P} \), denoted by \( N_{\hat{P}} \). Find a set \( S \subseteq N_{\hat{P}} \) of points with FA value close to \( P \), \( S = \{ M_i | M_i \in N_{\hat{P}}, f(P) - 0.3 \leq f(M_i) \leq f(P) + 0.3 \} \).
4. Obtain \( AV_2(M_i) \) from \( GF_2 \) for all \( M_i \in S \).
5. Evaluate the metric \( \text{dist}(AV_1(P), AV_2(M_i)) \) as given in the equation 2. The point \( M_i \) corresponding to the least distance is the corresponding point \( P' \).

4. RESULTS AND DISCUSSION

We have applied our matching paradigm to simulated and real tensor data.

**Simulated data** : We simulated a patch of tensors in Fig. 3(a) and rotated them to obtain Fig. 3(b). We then applied our approach to these patches. Exact correspondences were established between (a) and (b) supporting rotational invariance. Fig. 3(c) shows the colormap of correspondence of a point \( P \) in (a) with point \( P^s \) in (b), with closer points represented by shades of blue and those further away in tones of red. Fig. 3 (d) - (g) are the responses of applying the real and imaginary parts of the filter to a tensor patch around point \( P \). It can be seen from these responses that the imaginary parts of the Gabor filters act as edge detection filters and consolidate the orientation and the real part acts as a smoothing filter which accumulates information from all tensors around the voxel under consideration. Also as the scale decreases from (d) to (g), the information becomes more localized.

**Real Data** : We then applied our approach to human brain images. In Fig. 4, (a) is the template brain on which the
5. CONCLUSION AND FUTURE WORK

This paper presents a novel feature-based method of matching two DTI images. Gabor features are used as attribute vectors to characterize each voxel of the DTI image. The features are evaluated at multiple frequencies and scales thereby producing a complete coarse to fine characterization of the DTI data. We propose to refine the attribute vectors further by computing it in three dominant directions instead of just one. Also, we are extending this method to develop a method for registration of tensors.

6. REFERENCES