Optical Diffusion Tomography with Large Data Sets

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Optical Tomography
Inverse problem

Problem: Given measurements from multiple source-detector pairs, reconstruct the spatial distribution of the optical absorption and scattering (or “diffusion”) coefficients.

- Ill-posed
- Nonlinear
Size of the Data Set and Complexity of the Problem

\[ \phi_{ij} = \sum_j \Gamma_{ij,n} \delta \alpha_n \]

\[ i = 1, \ldots, N_s \]
\[ j = 1, \ldots, N_d \]
\[ n = 1, \ldots, L^3 \]

- \( N_s \) - number of sources
- \( N_d \) - number of detectors
- \( N = N_s N_d \) – total number of data points
- \( L^3 \) – number of volume elements
First generation Penn Scanner (~1995)

- Pulsed laser: 1 mW, 5MHz, 780, 830 nm
- Optical switch
- Detector module
- Router: 8 x 1
- Attenuator
- Reference channel

~100 source-detector pairs
Philips Scanner (~1998)

~10^5 source-detector pairs
Noncontact Imager (2005)

$10^8 - 10^{10}$ source detector pairs
Analytical vs Numerical Image Reconstruction Methods

### Advantages

<table>
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<tr>
<th><strong>Analytical</strong></th>
<th><strong>Numerical</strong></th>
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<tr>
<td>Arbitrarily high volume discretization</td>
<td>Generality</td>
</tr>
<tr>
<td>Computational efficiency</td>
<td>Large dynamic range of detectors is not required</td>
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### Disadvantages

<table>
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<tr>
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<tbody>
<tr>
<td>Requires special geometry, very large field of view, and high dynamic range of detectors…</td>
<td>Difficult to achieve high volume discretization</td>
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<tr>
<td></td>
<td>Large computational complexity</td>
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This talk is about numerical methods
Linearization ($D=\text{const}$)

\[
I_0(r_d, r_s) = C_d(r_d)C_s(r_s)(1 + \ell^*/\ell)^2 G_0(r_d, r_s)
\]

\[
I(r_d, r_s) = C_d(r_d)C_s(r_s)(1 + \ell^*/\ell)^2 G(r_d, r_s)
\]

\[
I(r_d, r_s)/I_0(r_d, r_s) = G(r_d, r_s)/G_0(r_d, r_s)
\]

Mean-filed approximation for $G$

\[
G(r_d, r_s) = G^2_0(r_d, r_s)/[G_0(r_d, r_s) + \int G_0(r_d, r)\delta\alpha(r)G_0(r, r_s) d^3r]
\]

\[
\int G_0(r_d, r)\delta\alpha(r)G_0(r, r_s) d^3r = \phi(r_d, r_s)
\]

$G$ – Green's function for the diffusion equation

$C$ – coupling coefficients

$I$ – measured intensity

$G_0$ – Green's function for the diffusion equation

$\phi$ – measurable data function
Discretization

\[ \Gamma \delta \alpha = \phi \]

- $N \times M$ matrix
- Vector of length $N$
- Vector of length $M$

$N \gg M$
Size of the Problem

• 20,000 sources per detector
• 29x29=841 source
• $N=1.7e7$ data points
• $M=15x51x51=3.9e4$ volume voxels
Computational complexity

SVD: \[ \delta \alpha^+ = \Gamma^+ \phi \]

\[ \Gamma^+ = (\Gamma^* \Gamma)^{-1} \Gamma^* \]

SVD complexity: \[ aM^2 N + bN^3 \]

\[ \delta \alpha^+ = (\Gamma^* \Gamma)^{-1} (\Gamma^* \phi) \]
Idea

• Compute the “backprojection” \( \Gamma^* \phi \) exactly
• Compute the “filter” \( (\Gamma^* \Gamma)^{-1} \) approximately
• This is not the same as binning (presumably, better)
• Allows to average out noise in the data (assuming the noise is non-correlated)
• Approximate method is compared to a calculation with an exact “filter”
Fast Matrix-Matrix Multiplication

\[ \Gamma \ast \Gamma = \sum_i B_i^* B_i \]

24GFlops at 4 Itanium-2 CPUs

BLAS-3 (DGEM) for each block, executed in parallel
Reconstruction of two black metal balls suspended in a 5cm thick plane-parallel tank with intralipid solution.

Diffuse wavelength: Approximately 10cm.

Field of view: 14cm x 14cm.

Central slice shown.

Ball diameter: 8mm.

Distance between balls: 30mm.

(reconstructed correctly)
Fifteen slices drawn parallel to the slab surface at equal separations

2.5mm between slices
Noise in the Data

Experimental data
Theoretical fit

$k_d = 0.58 \text{cm}^{-1}$
$\ell = 7 \text{mm}$

Absolute Error [CCD Counts]

CCD Pixel Number [1 pixel = 0.65mm]
(Spatial) Fourier Spectrum of the Transmitted Intensity

$h$ is the step on the surface of the slab corresponding to 1 CCD pixel

$(h = 6.5\text{mm})$
Conclusions

• It is possible to compute numerical SVD with > 1e7 data points (more than 2 orders of magnitude more than currently being used)

• This computation can be very significantly accelerated by the application of approximate numerical procedure discussed in this talk

• The potential benefits of such large data sets are
  (i) Higher spatial resolution
  (ii) Better noise tolerance

• However, with our current apparatus, these advantages can not be confirmed due to limitations specific to our experiment…
Note on Analytical Methods:

• Not reported in this talk
• MUCH faster
• Similar image quality
• A paper is in press in Optics Letters (2005)

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