SINGLE-SCATTERING OPTICAL TOMOGRAPHY

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Zero Scattering Regime:
Conventional X-Ray Tomography
Strong Scattering Regime: Diffuse Optical Tomography

- Many source-detector pairs
- Severely ill-posed IP
- Nonlinear IP
Mesoscopic Scattering Regime: Single-Scattering Tomography
SSOT And Other Modalities

<table>
<thead>
<tr>
<th></th>
<th>Linear IP</th>
<th>IP is MILDLY Ill-posed</th>
<th>Single projection</th>
<th>Reflection geometry</th>
<th>Nonionizing radiation</th>
<th>Quantitatrive images</th>
<th>New contrast mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-ray CT</td>
<td>Y (if $E=\text{const}$)</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>DOT</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>?</td>
<td>Y</td>
</tr>
<tr>
<td>SSOT</td>
<td>Y</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>
The Broken-Ray Integral Transform

(a) $\mu_s = \text{const} \ (\text{and is known})$

$$\int_{\text{SSR}(r_1,\hat{s}_1; r_2, \hat{s}_2)} \mu_t[r(\ell)]d\ell = \phi(r_1, \hat{s}_1; r_2, \hat{s}_2)$$

(b) $\mu_s \neq \text{const} \ (\text{and is unknown})$

$$\int_{\text{SSR}(r_1,\hat{s}_1; r_2, \hat{s}_2)} \mu_t[r(\ell)]d\ell - \ln \left[ \frac{\mu_s(R_{12})}{\langle \mu_s \rangle} \right]$$

$$\phi(r_1, \hat{s}_1; r_2, \hat{s}_2) = \phi(r_1, \hat{s}_1; r_2, \hat{s}_2)$$

The measurable data function:

$$\phi(r_1, \hat{s}_1; r_2, \hat{s}_2) = -\ln \left[ \frac{r_{12} \sin \theta_1 \sin \theta_2}{\langle \mu_s \rangle A(\hat{s}_1, \hat{s}_2)} \frac{I_{\text{measured}}}{I_{\text{incident}}} \right]$$
SIMULATIONS

• Forward model based on the RTE
• Isotropic scattering
• FULL ACCOUNT OF MULTIPLE SCATTERING
• BOUNDARY CONDITIONS SATISFIED EXACTLY
• 3D integral equation for density discretized on a rectangular grid
• Direct inversion of a well-posed square matrix
• Mathematical details on next page...
\[ [\hat{s} \cdot \nabla + \mu_a(r) + \mu_s(r)] I(r, \hat{s}) = \mu_s(r) \int \frac{1}{4\pi} I(r, \hat{s}') d^2\hat{s}' \]

\[ \mu_t(r) \]

\[ A(\hat{s}, \hat{s}') = \frac{1}{4\pi} = \text{const} \]

\[ u(r) = \int I(r, \hat{s}) d^2\hat{s} \]

\[ u(r) = u_b(r) + \int g_b(r, r') \frac{\mu_s(r')}{4\pi} u(r') d^3 r' \]

\[ I(r, \hat{s}) = I_b(r, \hat{s}) + \int G_b(r, \hat{s}; r', \hat{s'}) \frac{\mu_s(r')}{4\pi} u(r') d^3 r'd^2\hat{s}' \]

\[ G_b(r, \hat{s}; r', \hat{s'}) = g_b(r, r') \delta(\hat{u}(r - r') - \hat{s}') \delta(\hat{s} - \hat{s'}) \]

\[ g_b(r, r') = \int G_b(r, \hat{s}; r', \hat{s'}) d^2\hat{s} d^2\hat{s'} = \]

\[ = \frac{1}{|r - r'|^2} \exp \left[ - \int_0^{l(r - r')} \mu_t (r' + \ell \hat{u}(r - r')) d\ell \right] \]
Total attenuation $\mu_i$ is reconstructed in each slice.
\[ \mu_s h = 0.04 \]
\[ \mu_s L_z = 1.6 \]

ABSORPTION
Background:
\[ \mu_a h = 0.01 \]
Targets:
\[ 0.06 < \mu_a h < 0.2 \]
<table>
<thead>
<tr>
<th>Model</th>
<th>$n = 0$</th>
<th>$n = 1%$</th>
<th>$n = 3%$</th>
</tr>
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</table>

\[
\mu_s = 0.16h^{-1}
\]

\[
\mu_s L_z = 6.4
\]
Reconstruction of scattering+absorption in a thicker sample: $L_x = 25h$

34 source beams per slice

$L_y = 122h$

$L_x = 25h$

$L_z = 40h$

34 detectors per source at this angle
Simultaneous reconstruction of absorption and scattering

**SCATTERING**
Background: \( \bar{\mu}_s L_z = 1.6 \)
Targets: \( 1.33 \mu_s \leq \mu_s \leq 2 \mu_s \)

**ABSORPTION**
Background: \( \bar{\mu}_a = 0.1 \bar{\mu}_s \)
Targets: \( 2 \mu_a \leq \mu_a \leq 5 \mu_a \)
Stronger scattering inhomogeneities

**SCATTERING**
- Background: $\overline{\mu_s L_z} = 1.6$
- Targets: $2\overline{\mu_s} \leq \mu_s \leq 3\overline{\mu_s}$

**ABSORPTION**
- Background: $\overline{\mu_a} = 0.1\overline{\mu_s}$
- Targets: $2\overline{\mu_a} \leq \mu_a \leq 5\overline{\mu_a}$
**Stronger absorption (overall)**

**SCATTERING**
- Background: $\bar{\mu}_s L_z = 1.6$
- Targets: $2\mu_s \leq \mu_s \leq 3\mu_s$

**ABSORPTION**
- Background: $\bar{\mu}_a = \bar{\mu}_s$
- Targets: $2\mu_a \leq \mu_a \leq 5\mu_a$
Same as before, but larger optical depth

### SCATTERING

- **Background:** $\overline{\mu_s L_z} = 3.2$
- **Targets:** $2\mu_s \leq \mu_s \leq 3\mu_s$

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<th>$n=3%$</th>
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<tbody>
<tr>
<td><img src="image1" alt="Scattering Model" /></td>
<td><img src="image2" alt="Scattering Model" /></td>
<td><img src="image3" alt="Scattering Model" /></td>
<td><img src="image4" alt="Scattering Model" /></td>
</tr>
</tbody>
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### ABSORPTION

- **Background:** $\overline{\mu_a} = \overline{\mu_s}$
- **Targets:** $2\mu_a \leq \mu_a \leq 5\mu_a$

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<tr>
<td><img src="image5" alt="Absorption Model" /></td>
<td><img src="image6" alt="Absorption Model" /></td>
<td><img src="image7" alt="Absorption Model" /></td>
<td><img src="image8" alt="Absorption Model" /></td>
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SUMMARY

• SSOT allows accurate quantitative reconstruction of the attenuation function.
• With additional measurements, scattering and absorption can be reconstructed separately.
• Ill-posedness of the inverse problem is very mild.
• Tomographic imaging is feasible up to about six scattering lengths, with the noise-to-signal level of about 3% or less.
Preliminary Experiment
Figure 6: Experimental measurements of the specific intensity as a function of the exit position on the slab surface for intralipid concentrations 0.02% (a) and 0.04% (b).